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This report describes the research effort carried out under AFOSR Grant #86-0121. Two new unified theoretical-computational methods for analyzing unsteady aerodynamics involving rapid large amplitude motion of lifting bodies are described. One method is based on the zonal solution of the full Navier-Stokes equation and the other method is a simplified zonal method. Selected results are presented for two-dimensional rigid, articulate and flexible lifting bodies. Preliminary results for three-dimensional problems are also presented.

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STUDIES IN NON-LINEAR UNSTEADY AERODYNAMICS

(AFOSR GRANT NO. 86-0121)

FINAL REPORT

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March, 1989

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1. INTRODUCTION

This report reviews a research project carried out at Georgia Tech under the support of the Air Force Office of Scientific Research. This project was initiated in February of 1982 under AFOSR Grant #82-0108. In April of 1986, the grant number was changed to #86-0121. The original title of the project, "Studies in Non-Linear Unsteady Aerodynamics", as well as the original objectives of the project were retained under the new grant number. The research period covered by the present report is extensive. The scope of the research reviewed is broad. In addition to the principal investigator, a number of graduate students, post-doctoral fellows, research engineers and visiting scientists have contributed to the present program. Major results and conclusions have been published in a series of reports, Ph.D. theses, conference proceeding papers, journal articles, and chapters in specialist books. Chronological lists of major publications and Ph.D. theses prepared under the present project up to April of 1988 are provided in Reference 1 together with a description of research accomplishments that, in the opinion of the principal investigator, represent new and important contributions to the field of unsteady aerodynamics. Reference 1 is appended to the present report. The present report is then focused on the remaining period of the present program, between May 1988 and March 1989, partly under a non-cost extension of Grant #86-0121. For easier reference to Reference 1, the present report is organized in the same format as that of Reference 1, with parallel sections and headings.

The present project is a part of a broader and longer-ranged program of research in unsteady aerodynamics at Georgia Tech. The ultimate goals of this broader program are (1) to establish a body of knowledge and a precise understanding of the complex interactive fluid dynamic phenomena which dominate the aerodynamics of unsteady flows in the non-linear domain and (2) to establish a rational framework for the prediction of large aerodynamic forces generated as a result of large amplitude unsteady motions of lifting bodies.

Prior to the initiation of the present project in 1982, several unconventional concepts in aerodynamic analyses were conceived by Georgia Tech researchers. These concepts were shown, through analyses and numerical illustrations, to be ideally-suited for unsteady aerodynamic analyses under general circumstances, including those where large unsteady separated regions are present. The present project was planned to fully develop these concepts, to establish procedures that are capable of accurately predicting unsteady aerodynamic performance on a routine basis, and to utilize these procedures in studies of various non-linear unsteady aerodynamic problems. These objectives are known to be extremely challenging even today. They would have been overly ambitious in 1982, when the present program was formulated, in the absence of the unconventional concepts.

In planning the present research, the great challenges as well as the extensive and persistent efforts required to fully develop, calibrate and apply the unconventional concepts were fully recognized. In particular, it was realized that the new predictive methods to be established represent drastic departures from familiar methods. The tasks to be accomplished include not only studies of new problems of practical interest that are formidably difficult but also the creation of new analytical tools that transcend the well-known limitation of previous methods. In view of this reality, the present research was originally planned in three phases, each envisioned to be four to five years in duration. The first phase of the project was designed to demonstrate the practicality of utilizing the unconventional concepts in unsteady aerodynamic analyses in problems involving two spatial dimensions. Exploratory studies of several problems of practical interest were selected for this phase. The second phase of the project was originally planned to fully establish new two-dimensional predictive methods and to produce a reasonably broad information base about two-dimensional unsteady aerodynamic problems. The third phase of the project was planned as the concluding phase. During this phase, three-dimensional predictive methods were to be established and pilot three-dimensional problems of practical importance were to be studied using this method.

Under Grant #82-0108, research opportunity was fully provided to complete the first phase of the present project. The second phase of the project was carried out under Grant #86-0121. Research progress was substantially more rapid than anticipated. As a result, the third phase of the project, initially envisioned to be a longer-termed effort, was initiated under Grant #86-0121. At the conclusion of Grant #86-0121, several important issues related to three-dimensional applications of the unconventional concepts, which proved tc be remarkably successful in two-dimensional unsteady aerodynamics analyses, have been identified. Methods for resolving several long-standing major obstacles to three-dimensional non-linear unsteady aerodynamic analyses have been envisioned. It was anticipated that additional efforts of reasonable scope would firmly establish these methods as practical tools in three-dimensional aerodynamic analyses. It was then decided to carry out these additional efforts at no additional costs to AFOSR and to include the additional results in the final report. These additional efforts have been carried out. The time span required for these efforts, however, turned out to be substantially longer than anticipated because of the new and competing responsibilities the present researchers are required to take on in the absence of additional AFOSR funding. Partly for this reason, the submission of the present report is very much delayed.

2. UNIFIED THEORETICAL - COMPUTATIONAL METHODS

The present research project contains a theoretical (non-computational) component and a computational component. During the earlier part of the present research, these two components of the research were carried out as two separate efforts. During the course of the research, however, it was established conclusively that these two components of the research are fully compatible to one another. This compatibility permitted a close interaction between the theoretical and the computational aspects of the research. Through this theoretical-computational interaction, a much improved understanding of the physical processes important in the generation of large unsteady aerodynamic forces has been established. This understanding, in turn, has been utilized to develop unified theoretical-computational methods ideally suited for aerodynamic analyses involving large unsteady forces. Using these methods, extensive results have been obtained for many important unsteady aerodynamic problems involving two-dimensional flows. Results have also been obtained for selected problems involving three spatial dimensions.

It has been shown that, through the use of the concept of fundamental solution of partial differential equations, the time-dependent Navier-Stokes equations can be reformulated as an integro-differential equation set. Under general circumstances, this equation set can be used to construct a Navier-Stokes zonal method possessing the following distinguishing attributes: (1) the solution field can be confined to the viscous region of the overall flowfield and (2) the several viscous flow zones that co-exist in the flow can be solved independently without iteratively matching the different zones. These two unique attributes have been fully exploited under the present project for two-dimensional non-linear unsteady aerodynamic analyses. A computer code that permits two-dimensional viscous time-dependent flows, including those containing large separated regions, to be solved routinely and accurately have been prepared. This computer code is extremely efficient. The computation cost required to solve complex problems of practical importance is very reasonable. The code has been installed not only on various mainframe computers but also on personal computers. Several research groups in other universities, government and industrial laboratories have already used this code to study unsteady aerodynamic problems of interest to these groups. For three-dimensional applications, a computer code has been prepared and used to obtain results for selected problems of interest. This code in its present form does not incorporate all the advantageous features offered by the unified theoretical-computational approach. Further refinements of the three-dimensional code, however, are beyond the scope of the present project and are being pursued separately.

The present research has demonstrated that there exists a broad range of circumstances under which the flow Reynolds number is high and large unsteady aerodynamic forces are produced in the absence of significant recirculating flow zones. The present research has also shown that under these circumstances, drastic simplifications in unsteady aerodynamic analyses are permissible using the unified theoretical-computational approach. A simplified zonal method has been established under the present project for these circumstances. This simplified zonal method is remarkably effective in revealing the underlying physical processes responsible for the generation of large unsteady aerodynamic forces. The method accurately predicts these forces while requiring minimal amounts of computation. Furthermore, theoretical (non-computational) closed form solutions can be obtained for many important problems. Under the present project, the theoretical method and the simplified zonal method have been fully established not only for rigid bodies undergoing large amplitude unsteady motions but also for non-rigid (flexible and articulate) bodies.

Additional information about the unified theoretical-computational methods has been presented in Section 2 of Reference 1, which is appended to the present report, as well as in publications listed under Section 5 of the present report and of Reference 1.

3. ILLUSTRATIVE RESULTS

In Section 3 of Reference 1, which is appended to the present report, selected results using the unified theoretical-computational methods are presented. The results are principally for two-dimensional problems involving flows past solid bodies undergoing large amplitude rapid time-dependent motions. The solid bodies may be either rigid or non-rigid (deformable or articulate). In addition, preliminary results for a pilot three-dimensional study are also presented in Reference 1.

In the present report, brief discussions are presented for the vortex-airfoil encounter problem, which was previously discussed in Reference 1 under the heading of rigid two-dimensional bodies. References are then made to research completed subsequent to the period covered in Reference 1, including recent progress in studies of three-dimensional problems.

3.1. Rigid Two-Dimensional Bodies

Three problems of practical interest involving rigid two-dimensional lifting bodies have been studied. These are (1) the vortex-airfoil encounter problem, (2) the rapidly pitched airfoil problem and (3) the oscillating airfoil problem. Each of the three problems is important in a particular realm of unsteady aerodynamics. At the time Reference 1 was prepared, a Ph.D. thesis dealing with the vortex-airfoil encounter problem was complete. A theses dealing with the rapidly pitched airfoil problem and the oscillating airfoil problem was completed more recently (see Section 5). These theses present vastly more research results then described in the present report and in Reference 1.

Vortex-Airfoil Encounter

The vortex-airfoil encounter problem, in addition to being important to several well-known applications, is of fundamental importance in the understanding of physical mechanisms responsible for the production of large unsteady aerodynamic forces. When such forces are experienced, the presence and movement of a vortex are always manifest. This vortex may either be a passing vortex or a "separation vortex" formed and then shed into the wake. The interaction between the vortex and the airfoil is similar for the two cases. Under the present project, the case of a passing vortex is studied. This passing vortex may induce massive flow separation on the airfoil. This type of encounter is designated a "detached encounter". An encounter that does not induce massive flow separation is designated an attached encounter. Both types of encounters have been studied extensively.

The major accomplishments of the present research are

(1) For attached encounters, theoretical closed-form solutions have been obtained.

(2) For attached encounters, the simplified zonal method yields highly accurate results while requiring minimal amounts of computation.

(3) It has been demonstrated conclusively that attached encounters can lead to large unsteady aerodynamic forces.

(4) The Navier-Stokes zonal method has been shown to yield highly accurate results for both detached and attached encounters for both laminar and turbulent flows. Only a few minutes of computer time is required on the current generation mainframes to solve a typical case.

(5) The unified theoretical - computational approach developed under the present project has been shown to be ideally suited for non-linear unsteady aerodynamics. With this approach, various contributors to the unsteady aerodynamic loads can be identified and their relative importance assessed. For the vortex-airfoil encounter problem, the important contributors have been shown to be the unsteady boundary layer, the wake and the trajectories of the passing vortex.

(6) A comprehensive understanding of the physical mechanisms responsible for the production of unsteady aerodynamic force has been established for two-dimensional problems.

(7) The dependency of the unsteady aerodynamic load on various geometric and motion parameters, including the freestream velocity, the strength of the passing vortex, the passing distance between the vortex and the airfoil, the thickness and the camber of the airfoil, has been determined.

Rapidly Pitched and Oscillating Airfoils

Both the problem of the rapidly pitched airfoil and the problem of the oscillating airfoil are of substantial practical interest. Definitive experimental results for these problems are available in the open literature. These two problems have been studied extensively under the present project. Selected results obtained prior to April of 1988 have been reviewed in Reference 1. Additional results are presented in the Ph.D. thesis of I. H. Turner, completed in the summer of 1988.

The major accomplishments of the present research for these two problems parallel those stated earlier for the vortex-airfoil encounter problem. The present theoretical-computational results are found to be in remarkably good agreement with available experimental data. Work carried out after April 1988 under the present project has focused on the synthesis of the substantial amount of information generated and on the documentation of the methods established. Detailed solution manuals have been prepared and made available to researchers external to Georgia Tech.

3.2 Non-Rigid Two-Dimensional Bodies

Two categories of non-rigid two-dimensional bodies have been studied extensively under the present project. The first category involves articulate lifting bodies. Under this category, the well-known biofluiddynamic problem of Weis-Fogh as well as the problem of oscillating posterior plate have been studied. The second category involves flexible (deformable) lifting bodies. Under this category, the problem of swimming of a fish has been studied.

Reference 1 presents a brief review of the work completed prior to April 1988. Additional results are presented in the Ph.D. thesis of J. Singh, completed in the latter part of 1988.

The non-rigid body problem is significantly more complex than the rigid body problem discussed earlier. The major accomplishments of the present project for the non-rigid body problem are similar to those stated earlier for the rigid body problem. In addition, it has been shown that under a reasonably broad range of circumstances, the simplified zonal method yields accurate results and offers a great deal of physical insight. Definitive experimental data obtained under rigidly controlled environments are scarce for non-rigid bodies. The results obtained under the present project, however, have been shown to agree well with available experimental data. As is in the case of the rigid body problem, work carried out after April 1988 under the present project for non-rigid bodies has focused on the synthesis of the substantial amount of information generated and on the documentation of the methods established.

3.3 Three-Dimensional Lifting Bodies

At the time Reference 1 was prepared, the development of solution methods for three-dimensional applications was in its initial stage. Preliminary results obtained at that time are presented in Reference 1. Recent progress in the development of three-dimensional solution methods are described in Reference 2. Several difficult issues have been resolved recently. In particular, an accurate method for the determination of vorticity values on three-dimensional solid boundaries has been established. It has been shown that this method leads to good overall solution accuracy. The figure presented in Section 7 shows the computed lift coefficient for a flat plate wing at 10° angle of attack over a range of aspect ratios. This figure exemplifies the good agreement between the present results and experimental data. Additional results are presented in Reference 2.

4. CONCLUDING REMARKS

Under the present research project, unified theoretical-computational methods for unsteady viscous aerodynamic analysis have been fully developed, documented and utilized in comprehensive studies of a broad range of two-dimensional problems. An extensive information base has been created using the unified methods. This information base has been synthesized and utilized to produce a comprehensive understanding of the important physical processes contributing to the generation of large unsteady forces. The methods have been extended to three-dimensional problems. The development of three-dimensional methods has not reached a stage of full maturity. Further refinements of the three-dimensional method, however, are beyond the scope of the present project and are being pursued separately.

A most significant conclusion of the present project is that highly complex non-linear unsteady aerodynamic problems need not be tackled through brute force computation alone. Under a broad range of circumstances important to non-linear unsteady aerodynamics, it is possible to accurately predict large unsteady aerodynamic forces using no computation or very little computation. This fact has been amply demonstrated by the success of the theoretical method and the simplified zonal method established under the present project. Under general circumstances where the Navier-Stokes equations need to be solved, the amount of computation required can also be reduced to a very moderate level through mathematical analyses and the viscous theory of aerodynamics. This fact has been conclusively demonstrated by the success of the Navier-Stokes zonal method.

The distinguishing features of the unified theoretical-computational methods permitted the establishment of an extensive information base as well as a comprehensive understanding of the important physical processes involved in the generation of non-linear unsteady aerodynamic forces in two-dimensional application. Such an information base and a comprehensive understanding are feasible also for three-dimensional applications. The massiveness of three-dimensional unsteady flow problems demands substantial additional research efforts. The experience of the present research suggests, however, that these important goals can be attained in the near future provided that adequate support and encouragement are available for persistent and uninterrupted research.

In reviewing the work carried out under the present project, the principal investigator recollected vividly the support and encouragement of Michael Francis, who served as the AFOSR monitor of this project during its initial phase. Michael provided the needed mix of interaction and latitude for the present researchers to pursue a subject which, because of its scope, its formidable difficulties and its fundamental nature, requires long-termed efforts at a vigorous but unharried pace. With current increasing emphasis on "focused" research and shortened "payoff" periods, opportunities for long-termed basic research efforts such as the present one are becoming rarer. For this reason, the research environment provided for the present project during its initial phase is today very much treasured by university researchers, this principal investigator included. It is thus with great satisfaction as well as a degree of nostalgia that this principal investigator carries out this final phase of effort under AFOSR support, reporting the work performed under this project.

5. PUBLICATIONS

Publications under AFOSR Support

See Section 5 (pages 40 to 42) of Reference 1 for Items #1 through #29.

30. Wang, C.M. and Wu, J.C., "Accurate Determination of Surface Pressure in High Reynolds Number Flows" Proceedings of 1st International Conference on Computational Method in Flow Analysis, September 1988.

31. Wu, J.C., "Zonal Solution of Time-Dependent Three-Dimensional Flow Problems" Proceedings of 7th International Conference of Finite Element Methods in Flow Problems, April 1989.

32. Hsu, T.M. and Wu, J.C., "Vortex Flow Model for the Blade-Vortex Interaction Problem", AIAA Journal, Vol. 26, No. 5, pp. 621-623, 1988.

33. Tuncer, I.H., Wu, J.C. and Wang, C.M. "Theoretical and Numerical Studies of Oscillating Airfoils", AIAA Paper #89-0021, January 1989, Accepted for publication in the AIAA Journal.

34. Wu, J.C. and Wang, C.M., "Recent Advances in Solution Methods for Unsteady Viscous Flows", Chapter 7 in Developments in Boundary Element Methods - 6, (P. K. Banerjee and L. Morino, Eds.), London: Elsevier Applied Science Publishers, in print (1990).

35. Wu, J.C., "Linkage Between Potential and Viscous Flows," International Symposium on Boundary Element Methods, Hartford, Connecticut, October 1989.

Ph.D. Dissertations Under AFOSR Support

1. H. Hu-Chen, "A Study of the Aerodynamic Performance of Weis-Fogh Wing," March 1985.

2. T.M. Hsu, "A Study of a Vortex-Lifting Surface Interaction Problem," March 1986.

3. M.Y. Sohn, "A Numerical Study of the Weis-Fogh Mechanism," August 1986.

4. J. Singh, "A Computational Study of the Vortex Trapping Phenomena," September 1988.

5. I. Tuncer, "Theoretical and Computational Studies of Dynamic Stall of Rapidly Pitched Airfoil," September 1988.

6. R. Reilman, "A Theoretical and Numerical Study of the Weis-Fogh Problem Including Wing-Separation," in preparation.

7. M. Patterson, "Zonal Computation of Compressible Navier-Stokes Equations," in preparation.

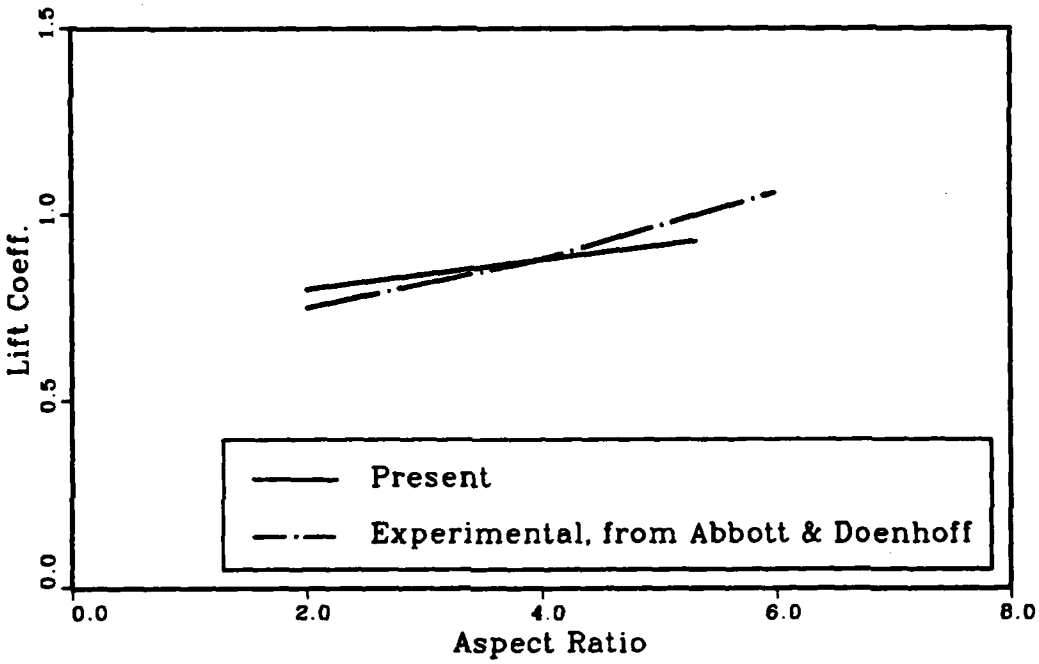
6. REFERENCES

1. Wu, J.C., "Studies in Non-Linear Unsteady Aerodynamics", A Technical Report submitted to AFOSR (Grant No. 86-0121), April 1988.

2. Wang, C.M. and Wu, J.C., "A Numerical Method for Three-Dimensional Viscous Flows", AIAA Paper #90-0236, January 1990.

See Section 6 of Reference 1 for additional literature pertinent to the present project.

7. FIGURE



Lift Variation of Wings, Angle of Attack 10°

APPENDIX

STUDIES IN NON-LINEAR UNSTEADY AERODYNAMICS

(AFOSR GRANT NO. 86-0121)

A TECHNICAL REPORT

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April 1988

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1. INTRODUCTION

In 1982, a research program in unsteady aerodynamics was initiated at the Georgia Institute of Technology under the support of the Air Force Office of Scientific Research. A primary objective of this program was to establish new methods for the routine and accurate prediction of large unsteady aerodynamic forces acting on aerodynamic surfaces undergoing rapid large-amplitude unsteady motions (Reference 1). The great challenges associated with such an ambition undertaking were fully recognized in 1982. Accordingly, this program was planned as a long-term effort to be carried out in many phases. Under the present research program, however, work has progressed at a pace much more rapid than originally anticipated. The first phase of this research was concluded in March of 1986. In view of the rapid progress made, the second phase, initiated in April of 1986, was envisioned to be the final part of the present research program (Reference 2). This second phase research was planned as a five-year effort. In the present report, progress made during the first two years of the second phase research is reviewed.

It is well-known that flowfields associated with the generation of large unsteady aerodynamic forces are inherently interactive, viscous, and non-linear. Massive flow separation can and often does occur. Previously available methods of aerodynamic analyses are not well-suited for the study of such flows. Prior to the initiation of the present research program, several new theoretical and computational concepts were conceived at Georgia Tech. The first phase of the present program was planned to investigate these new concepts and to demonstrate the practicality of applying these concepts in unsteady aerodynamic analyses through exploratory studies. Several pilot problems involving two-dimensional unsteady viscous flows were selected for investigation during this first phase research. The second phase of the present research was originally planned to establish practical predictive methods as well as to produce a reasonably broad information base about unsteady aerodynamic problems involving two spatial dimensions. This original scope of the second phase was more recently broadened to include studies of some pilot three-dimensional problems (Reference 2).

The present research program contains a theoretical (non-computational) component and a theoretical component. During the earlier part of the first phase research, these two components of the research were carried out as two separate efforts. During the course of the research, however, it was established conclusively that these two components of the research are fully compatible to one another. This compatibility permited a close interaction between the theoretical and the computational aspects of the research. During the reporting period, through this theoretical-computational interaction, a much improved understanding of the physical processes important in the generation of large unsteady aerodynamic forces has been established. This understanding, in turn, has been utilized to develop unified theoretical-computational methods ideally suited for aerodynamic analyses involving large unsteady forces. Using these methods, extensive results have been obtained for many important unsteady aerodynamic problems involving two-dimensional flows. Results have also been obtained for pilot problems involving three spatial dimensions.

Two outcomes of the present research are particularly noteworthy. First, the availability of the unified methods has made it possible to study various two-dimensional problems involving large unsteady aerodynamic forces routinely and accurately. These methods are highly efficient and they are useful for a broad range of applications. During the reporting period, substantial efforts have been devoted to the detailed documentation of these new methods. These documents have been requested by a large number of interested researchers outside Georgia Tech. Studies have been carried out during the reporting period to generalize these new methods for three-dimensional applications. It has been shown that no conceptual difficulties exist in such a generalization. Because of the greater complexity of three-dimensional flows, however, extensive additional efforts are required to produce a routine predictive capability for three-dimensional applications.

The second noteworthy outcome is that the extensive information acquired under the present program offered an unprecedented opportunity to establish a comprehensive understanding of the subject of unsteady aerodynamics. During the reporting period, efforts have been made to synthesize the information already acquired, including the most recently obtained results for three-dimensional flows. One goal of these efforts is to develop "rules of thumb" concerning the influences of the geometry and the motion parameter of lifting bodies on unsteady aerodynamic performance. This goal is of obvious importance in engineering applications involving the generation and utilization of large unsteady aerodynamic forces.

In June of 1987, a report (Reference 3) was prepared for the period April 1, 1986, to June 1, 1987. The present reporting period overlaps a part of the previous reporting period. For this reason, several parts of the present report duplicate contents of the previous report. As stated earlier, a primary objective of the present program is to establish new methods for unsteady aerodynamic analysis. The status of the new methods are reviewed in Section 2 of the present report together with brief descriptions of the underlying concepts. Selected results for various unsteady aerodynamic problems are provided in Section 3 to illustrate the appliction, the efficiency and the accuracy of the new methods. Extensive investigations of a number of important unsteady aerodynamic problems have been carried out under the present program. Most of the results of these investigations have been presented in publications listed in Section 5 and are not included in this report.

2. UNIFIED THEORETICAL-COMPUTATIONAL METHODS

During the reporting periods, new predictive methods for unsteady aerodynamics have been developed using a unified theoretical-computational approach. These new methods have been utilized extensively in studies of two-dimensional problems during the reporting period. The generalization of the unified methods for three-dimensional applications has been initiated. Additional efforts required to fully develop the unified methods for three-dimensional applications have been planned in detail and are in progress.

During the reporting period, additional familiarity with the unconventional concepts which form the basis of the inified theoretical-computational methods has been acquired. In the present section, the distinguishing features of these concepts are briefly reviewed. The present status of the unified methods is described.

2.1 Viscous Theory of Aerodynamics

A focal point of the present research program is a general viscous theory of aerodynamics (Reference 4) derived previously by the principal investigator. This theory is an exact consequence of the full viscous (time-dependent Navier-Stokes) equations. The theory relates the aerodynamics force and moment acting on lifting bodies (rigid, articulate, or flexible) to their vorticity environment. The mathematical statement of this theory for the total aerodynamic force $\vec{F}$ acting on lifting bodies present in a flow is

$$\vec{F} = -\frac{\rho}{d-1}\frac{d}{dt}\int_{R_\infty} \vec{r} \times \vec{\omega}\, dR + \rho \sum_{j=1}^{J} \left(\frac{d}{dt}\int_{R_j} \vec{v}_j\, dR\right) \qquad (1)$$

where ρ is the density of the fluid, t is the time coordinate, $\vec{r}$ is a position vector, $\vec{\omega}$ and $\vec{v}$ are respectively the vorticity vector and the velocity vector, d is the dimensionality of the problem, i.e., d = 2 and 3 respectively for two- and three-dimensional flows, R_∞ is the unlimited infinite region occupied jointly by the fluid and the solid and R_j is the region occupied by the jth body, there being a total of J individual bodies present in the fluid.

The first integral on the right side of Equation (1) is an integral of the first moment of vorticity. The integrals over R_j express the inertia effects of the solid bodies. With the motion of the solid bodies prescribed, the integrals over R_j are determined. A knowledge of the time-dependent distribution of vorticity therefore permits the aerodynamic force to be evaluated using Equation (1). An equation similar to Equation (1) is available for the moment of aerodynamic force.

In an aerodynamic flow involving a viscous fluid, there generally exists a flow region, surrounding and trailing the lifting body, in which the vorticity is non-zero and the viscous effects are important. Outside this viscous region, there exists a large potential region in which the vorticity, and hence also the viscous effects, are absent. Obviously, the potential region, because of the absence of vorticity, does not contribute to the integrals in Equation (1). Furthermore, the integral involving vorticity is replaceable by a sum of zonal integrals, each covering a specific viscous zone containing vorticity and each determinable individually. In consequence, aerodynamic analyses based on the viscous theory of aerodynamic possess two distinguishing attributes. First, they permit the analyses to be confined to the viscous region of the flow where vorticity is present. Second, they permit the contributions of the different viscous zones coexisting in the flow to be evaluated individually. Through these attributes, important contributors to non-linear unsteady aerodynamic load can be identified and their relative importance determined.

Once the time-dependent vorticity distribution associated with a lifting body is determined, Equation (1) permits the unsteady aerodynamic performance of the lifting body to be evaluated. Since this equation is an exact consequence of the full time-dependent Navier-Stokes equations, the accuracy achievable in the aerodynamic analysis is dependent only upon the accuracy of the information on the vorticity distribution. Under general circumstances, detailed computations of the unsteady flowfields are necessary in order to accurately determine the time-dependent vorticity distribution. The full Navier-Stokes equations need to be solved numerically for this purpose. In Section 2.2, the anatomy of the unsteady viscous aerodynamic flow under general circumstances is briefly described. In Section 2.3, it is shown that, in the computation of the time-dependent vorticity distribution, it is possible to confine the computation field to the viscous region of the flow under general circumstances. Indeed, the development of a full Navier-Stokes zonal method has been finalized during the reporting period. This zonal method not only permits the computation field to be confined to the viscous region, but it also allows the several viscous zones coexisting in the viscous region to be computed individually without iterative matching of the zones.

Under special circumstances discussed in Section 2.4, the time-dependent vorticity distribution in the viscous zone can be approximately evaluated and it becomes unnecessary to obtain numerical solutions to the full Navier-Stokes equations. The approximations are justifiable over a reasonably broad range of circumstances.

Studies of unsteady aerodynamics involving rigid two-dimensional lifting bodies using the viscous theory of aerodynamics were initiated during the early stage of the first phase of the present program. During the reporting period, these studies have been completed. Routine theoretical procedures have been established for analyzing these problems. Work has been carried out during the present reporting period to use the viscous theory of aerodynamics in studies of problems involving non-rigid bodies, including articulate bodies and flexible (deformable) bodies. Substantial experience and extensive results have been acquired during the reporting period for these problems (Reference 5). It is anticipated that these studies will be concluded and routine theoretical procedures for unsteady aerodynamic analysis involving non-rigid bodies will be fully established in the near future.

The viscous theory of aerodynamics is valid for three-dimensional problems. During the reporting period, experience has been gained in the use of this theory for three-dimensional unsteady aerodynamic analysis. A reasonable understanding of three-dimensional mechanisms contributing to unsteady lift, drag and moment has been developed. This effort is being continued.

2.2 Viscous Flow Zones

It is well known that vorticity (vortices) can neither be created nor be destroyed in the interior of an inviscid fluid domain. This well-known theorem of Helmholtz is also valid in a viscous fluid (Reference 4). All vorticity present in the interior of the fluid domain must originate from the fluid/solid interface. This vorticity, once generated, spreads into the interior of the viscous fluid by diffusion and, once there, is transported away from the boundary by both convection and diffusion. This process of vorticity generation and transport obviously produces a region of significant vorticity surrounding and trailing any aerodynamic body immersed in and moving relative to a fluid.

Since the effective rates of diffusive and convective transport of vorticity are finite and the vorticity present in a fluid must originate from the fluid/solid interface, the region of significant vorticity is finite in extent at any finite time level after the onset of a fluid motion. This region is surrounded by a region in which no significant vorticity is present. It is well known that viscous effects are important only in the region where vorticity is significant. The region absent of vorticity is an inviscid (potential) flow zone.

The viscous region is, in general, composed of attached boundary layer and detached flow zones. In two-dimensional flows, the detached flow zones contain recirculating flows (separation bubbles), wakes and starting vorticity systems. The vorticity in the detached flow zones originates from the boundary layers. That is, this vorticity is transported to the recirculating zone, the wake zone, etc., from the boundary layer zone. For three-dimensional flows, "tip vortices" are also present. The vorticity content of the tip vortices also originates form the boundary layers.

The length scales of the attached viscous zone, the detached viscous zone and the inviscid zone that coexist in the flow are drastically different from one another. For high Reynolds number flows, the diffusion process is much slower than the convection process. In consequence, in the attached zone, the vorticity generated at the fluid/solid interface spreads only a short distance away from the solid before it is carried away by convection. The boundary layers are therefore very thin. In comparison, the detached viscous zones are generally much thicker. The inviscid region is infinite in extent. The mathematical characters of the various flow zones present are also greatly different from one another (Reference 6). With previous methods of aerodynamic analysis, the simultaneous presence of these diverse lengths scales and diverse flow characteristics has led to serious theoretical and computational difficulties. These difficulties are removable if the viscous zones coexisting in the unsteady flow can be treated individually.

Under the first phase of the present research program, the theoretical and the computational components were made to reinforce one another. The close interplay of the two aspects produced a thorough familiarity with the flow features and physical processes important to unsteady aerodynamics in two dimensions. This familiarity is responsible for the rapid development, calibration, and application of the unified methods of two-dimensional unsteady aerodynamic analysis during the reporting period. A reasonable degree of familiarity has been established for three-dimensional applications during the reporting period. This familiarity is expected to facilitate the development of the unified methods for three-dimensional applications. Indeed, because of the great complexity of the three-dimensional unsteady viscous flow, a thorough familiarity with the important flow features and physical processes is a prerequisite to the development of any method that can accurately and routinely predict the unsteady aerodynamic performance of three-dimensional lifting bodies. Work completed thus far has identified several important issues needing additional research efforts. Several possible solution procedures have been conceived to deal with these issues. These procedures are undergoing systematic evaluation and calibration.

2.3 Navier-Stokes Zonal Method

The Navier-Stokes zonal method is based on the reformulation of the full viscous flow equations using the concept of fundamental solution of partial differential equations. During the present research program, a large number of different formulations were investigated. The results of these investigations are presented in various publications listed in Section 5 of this report. The following formulation, which partitions the overall flow problem into its kinematic and kinetic aspects, is valid for both three-dimensional and two-dimensional flows. Under the present research program, computer programs have been prepared and documented for routine computation of two-dimensional unsteady Navier-Stokes flows. Progress has been made towards the establishment of a routine computational capability for three-dimensional Navier-Stokes flows.

The kinematic aspect of the problem is expressed as an integral representation for the velocity field (Reference 6):

$$\vec{v}\,(\vec{r},t) = -\int_R \vec{\omega}_0 \times \vec{Q}_0 \, dR_0 + \oint_B \left[(\vec{v}_0 \cdot \vec{n}_0)\, \vec{Q}_0 - (\vec{v}_0 \times \vec{n}_0) \times \vec{Q}_0 \right] dB_0 \tag{2}$$

where B is the boundary of the region R, $\vec{n}$ is a normal unit vector on B directed outward from R, the subscript "o" indicates that a variable or an integration is in the $\vec{r}_0$ space, e.g., $\vec{\omega}_0 = \vec{\omega}_0(\vec{r}_0, t)$ and $\vec{Q}_0$ is the gradient of the fundamental solution of elliptic differential equations, and is given by,

$$\vec{Q}\,(\vec{r}, \vec{r}_0, t) = \frac{\vec{r}_0 - \vec{r}_0}{2(d-1)\pi \, |\vec{r}-\vec{r}_0|^d} \tag{3}$$

Equation (2) describes the kinematic relationship between the velocity and the vorticity fields. The kinetic aspect of the problem is described by the vorticity transport equation (Reference 6),

$$\frac{\partial \vec{\omega}}{\partial t} = \nabla \times (\vec{v} \times \vec{\omega}) = \nu \nabla^2 \vec{\omega} \tag{4}$$

In Equation (4), the terms on the right side describe the kinetic process of convection and diffusion.

Equations (2) and (4) are equivalent to the full time-dependent Navier-Stokes and continuity equations. In a computation procedure, a loop that advances the solution from one time level to another is composed of the following two major steps:

(a) The velocity and vorticity distributions at an old time level is placed into the right side of Equation (4). The time rate of change of the vorticity field is thus determined. This rate of change is used to compute the new vorticity distribution at a subsequent time level.

(b) The new vorticity distribution, along with prescribed velocity boundary conditions, are placed into the right side of Equation (2). Velocity values at the new time level are computed explicitly, one point of $\vec{r}$ at a time, through a numerical quadrature procedure.

With the time-dependent vorticity values computed, the unsteady aerodynamic forces can be evaluated using Equation (1).

It is obvious that each term in Equation (4) vanishes in the inviscid zone where the vorticity is zero. Therefore, the kinetic part of the overall computation procedure can be confined to the viscous zones. The velocity values can be computed explicitly, point by point, using the integral representation, Equation (2). The convection term in Equation (4) vanishes wherever the vorticity is zero. As aresult, velocity values need not be computed in the inviscid zone. Also, the integrand of the vorticity-moment integral is zero in the inviscid region. Therefore, the kinematic part of the computation can be also confined to the viscous region. In consequence, the overall computation can be confined to the viscous region.

It is important to note that the differential equations describing the kinematic aspect of the flow are elliptic. The ability to compute the velocity values [step (b) of the loop] explicitly, point by point, is not available if a finite-difference or a finite-element method is used to solve these differential equations. The ability to compute velocity values explicitly is available only if the differential equations are first recast into an integral representation, such as Equation (2). Without this ability, it is not possible to confine the solution field to the viscous region.

The integral representation for the velocity field, Equation (2), allows velocity values to be computed explicitly, one point at a time. This ability permits the velocity values on the boundary of a specific zone to be computed independently of all other computations. Once this boundary velocity computation is accomplished, the problem of computing the flow inside this zone can be carried out independently of the computation of all other zones.

The attributes of the Navier-Stokes zonal method to confine the computation field and to permit zonal computation is obviously fully compatible with those of the viscous theory of aerodynamics described earlier. During the reporting period, the Navier-Stokes zonal method has been fully developed and successfully synthesized with the viscous theory of aerodynamics for two-dimensional applications. Full documentation of the zonal method has been completed during the reporting period. A solution manual (Reference 7) which contains a description of the mathematical foundation of the method, a listing of the computer program, including all subroutines developed, as well as a sample solution illustrating the numerical procedure in detail, has been prepared. A large number of requests for this manual have been received from aerodynamicists interested in using the zonal method in their unsteady aerodynamics research.

2.4 Simplified Zonal Method

In most engineering applications, the flow Reynolds number is high and the attached boundary layers are thin. Under many circumstances, no significant recirculating flow zone is present and the vorticity in the wake zone is a continuation of the vorticity of the boundary layers. Drastic simplifications are permissible in aerodynamic analysis under these circumstances. The present research has shown that there exists a reasonably broad range of circumstances under which large unsteady aerodynamic forces are produced in the absence of flow separation. The simplified zonal analysis developed under the present program is for this range of circumstances and is of substantial practical interest.

In the absence of a recirculating flow, the wake vorticity takes on the following forms. In a two-dimensional flow, the vorticity in the boundary layer is shed form the vicinity of a lifting body's trailing edge and feeds into the wake. Since the boundary layer is thin, the wake is initially thin. The rate of growth of the wake thickness is slow since diffusion is a slow process. Thin wake layers, however, are unstable and they tend to roll up and form clusters of concentrated vortices. These forms of wake vorticity are referred to, for convenience, as wake layers. Wake layers are formed during intervals of slowly varying motions of lifting bodies. In steady flows, wake layers do exist. However, the integrated vorticity across the wake layer is zero. In consequence, in steady-state aerodynamic analyses, the presence of the wake layer is unimportant. In unsteady aerodynamic analyses, however, the presence of the wake layer needs to be evaluated.

In the case of an unsteady flow associated with a lifting body undergoing large amplitude rapid changes of motion, large amounts of vorticity are shed into the wake in short intervals of time. The shed vorticity appears not as a wake layer, but as concentrated dosages of vorticity. A well-known example of this phenomenon is the formation of the starting vortex as the angle of attack of an airfoil is changed quickly. Another example is the production of "leading edge vortices" during the fling phase of the Weis-Fogh wings (Reference 8). This form of wake vorticity, referred to in the present study as a Weis-Fogh vortex, is often present in flows associated with large unsteady aerodynamic forces. The Weis-Fogh vortices, in fact, often are prominent features in such flows.

Wake layers and Weis-Fogh vortices are present in three-dimensional flows as well as in two-dimensional flows. In three-dimensional lifting flows, tip or leading edge vortices are also present. The tip or leading edge vortices are also thin layers of vorticity representing continuations of boundary-layer vorticity. The orientation of the vorticity in the tip vortices, however, differs from that in the wake layer. Tip vortices do not quickly break up into clusters. Rather, they tend to roll up into twisted bundles.

The vorticity contents of the boundary layers are justifiably approximated by vortex sheets in aerodynamic analyses using the viscous theory of aerodynamics. This approximation neither implies the inviscid fluid idealization nor assumes boundary layer simplifications. The approximation merely recognizes the fact that, with Equation (1), the distribution of vorticity across the thickness of the boundary layers is not important in aerodynamic analyses since the thickness of these layers are very small. Clusters of vorticity such as the Weis-Fogh vortices and the rolled-up wake layers are justifiably approximated by a set of vortex filaments, i.e., point vortices in two-dimensional flows. The part of the wake layer that is exists as a true layer is also justifiably approximated by a set of vortex filaments. This is permissible since the wake layer is formed only during time intervals of slowly varying motion of the lifting body. The vorticity content of the wake layer is not large. Errors introduced by the vortex filament approximation are small and acceptable.

For two-dimensional flows, tip vortices are absent. The validity of the approximations of the vorticity content of the boundary layer, the wake layer, and the Weis-Fogh vortices in high Reynolds number flows have been thoroughly calibrated during the reporting period through extensive analyses and numerical illustrations. It has been shown that the process of formation of the nascent wake vortex is an important issue in unsteady aerodynamic analysis. Under the present research program, a highly accurate method for the evaluation of the strength and location of the nascent vortex has been fully established. This method is based on an analysis of the viscous boundary-layer flow and removes the uncertainty of previous methods based on the inviscid fluid assumption. The development of this method for unsteady flows involving rigid lifting bodies was carried out prior to the present reporting period. During the reporting period, this method has been generalized for non-rigid lifting body applications. It has been shown conclusively through analyses and numerical illustrations that the method correctly simulates the viscous process of unsteady wake formation and yields accurate aerodynamic results. A journal article describing this method has been prepared and is schedule to appear soon (Reference 9) in the open literature.

With the approximation just described, Equation (1) is expressible as (Reference 5)

$$\vec{F} = -\frac{\rho}{d-1}\frac{d}{dt}\oint_S \vec{r} \times (\vec{\gamma} + \vec{v}\times\vec{n})ds - \frac{\rho}{d-1}\frac{d}{dt}\sum_k(\vec{r}_k \times \vec{\Gamma}_k) \tag{5}$$

where S is the surface of the lifting body, $\vec{r}_k$ and $\vec{\Gamma}_k$ are the location and the strength, respectively, of the kth wake and Weis-Fogh vortex, and $\vec{\gamma}$ is the strength of the vortex sheet representing the vorticity in the boundary layer. $\vec{\gamma}$ is defined by

$$\vec{\gamma}(s) = \int_0^\delta \vec{\omega}\ (s,n)\ dn \tag{6}$$

where s and n are the tangential and normal boundary layer coordinates, respectively, and δ is the thickness of the boundary layer.

In References 5 and 10, it is shown that if $\vec{r}_k$ and $\vec{\Gamma}_k$ are known at any instant of time, the vortex sheet strength $\vec{\gamma}$ can be determined using a panel method. For many important two-dimensional unsteady aerodynamic problems, closed form analytical expressions have been obtained for $\vec{\gamma}$. These expressions relate $\vec{\gamma}$ to the shape and motion of the lifting body, which may be either rigid or non-rigid, and to the wake parameters. These analytical expressions, when placed into Equation (5), yield closed form solutions for the unsteady aerodynamic force. These closed-form solutions offer a great deal of physical insight to the physical process of generating large aerodynamic forces. The influence of the shape of the lifting body, rigid or non-rigid, and that of the motion parameters are unraveled through these closed form solutions. Obviously, for engineering applications involving large unsteady aerodynamic forces, these solutions are valuable resources in the design process.

For three-dimensional flows, tip vortices are justifiably approximated by bundled vortex filaments. Work has been initiated during the reporting period to utilize this approximation, together with the approximations already described for two-dimensional applications, in the establishment of a highly efficient procedure for unsteady aerodynamic analysis involving large amplitude motions of three-dimensional lifting bodies. Substantial experience has been acquired during the reporting period. The practicality of generalizing the simplified zonal method for three-dimensional applications has been conclusively established.

3. ILLUSTRATIVE RESULTS

In the present section, selected results using the unified theoretical-computational methods are provided. Three categories of practical problems have been investigated thus far. These are: (1) two-dimensional rigid bodies undergoing large amplitude rapid time-dependent motions, (2) two-dimensional non-rigid bodies undergoing large amplitude rapid time-dependent motions, and (3) three-dimensional flows associated with wings of finite span. Results for the two-dimensional problems represent end products of several years of intensive work. They demonstrate the high level of accuracy presently achieveable using the unified methods. In terms of computer time requirements, two-dimensional Navier-Stokes results are typically obtained using less than 20 minutes of CPU time on Cyber 990, a relatively slow machine by today's computation standards, for each case presented. Since this means that, on a super computer, only a few minutes are needed for each case, parametric studies needed for design purposes can be carried out economically. The results presented here also show that the simplified analysis for two-dimensional problems yields accurate results under a reasonably broad range of circumstances under which large unsteady aerodynamic forces are generated. The amount of computation required using the simplified anlysis is minimal. Theoretical (non-computational) closed form results have been obtained for each of the two-dimensional problems investigated. These closed form results provide a great deal of physical insight. They also permit the effects of various geometric and motion parameters on the unsteady aerodynamic performance to be determined directly. In contrast, the development of three-dimensional predictive methods have not yet reached a stage of maturity. Three-dimensional results presented here are for a pilot study carried out recently.

3.1 Rigid Two-Dimensional Bodies

Three problems involving rigid two-dimensional lifting bodies have been investigated. Each of the three problems has been studied using three different methods: (1) the theoretical method; (2) the simplified zonal analysis; and (3) the full Navier-Stokes zonal method. As discussed in

Section 2, the last two methods are unified theoretical-computation methods which employ the viscous theory of aerodynamics as well as zonal procedures of computation. The third method is valid under general circumstances. Flows under both laminar and turbulent conditions have been investigated using this method. The second method utilized simplifying approximations that are valid under a reasonably broad range of important circumstances. The first method is non-computational and has a more restricted range of applicability.

There are several reasons for investigating these three problems repeatedly using three different predictive methods. A primary goal of the present research, as stated earlier, is to develop new methods well-suited for unsteady aerodynamic analysis. The different studies reinforce each other and serve as an intermediary for the systematic development, calibration, and refinement of each method. Each method, with its distinguishing attributes, is ideally suited for a specific spectrum within the broad, complex and difficult scope of unsteady aerodynamic phenomena. The availability of all three methods in their mature stage of development is important in this challenging field of research.

Each of the three problems investigated is important in a different realm of unsteady aerodynamics. The prominent physical phenomena as well as the specific applications connected with each of the three problems are all important in engineering efforts. During the course of the present research, extensive results has been obtained for each problem. some of these results have been presented in the open literature. In the present report, only selected recent results are presented. The primary purpose of presenting these results is to demonstrate the mature stage of development of the methods and to bring into focus several accomplishments judged significant by this principal investigator.

a. <u>Vortex-Airfoil Encounter</u>

The problem of unsteady aerodynamic force acting on a lifting body induced by a passing vortex is important to a large number of well-known applications. This problem is also of fundamental importance in the understanding of physical processes that lead to the generation of large unsteady aerodynamic forces since, whenever such forces are experienced by a lifting body, the presence and movement of a relatively strong vortex are always manifest. This strong vortex may either be a passing vortex produced by a preceding lifting body or a "separation" vortex which is formed and then shed into the wake. The necessary involvement of the strong vortex in the generation of the large forces is unraveled by the viscous theory of aerodynamics.

The vortex-airfoil encounter problem has been comprehensively studied by T.M. Hsu (Reference 11) in his Ph.D. research. The three methods stated earlier each yielded useful practical results as well as valuable physical insight. Results obtained using these three methods have been synthesized and used in the further development of unified theoretical-computational methods for more complex applications.

Highlights of the vortex-airfoil encounter study are summarized below:

(1) Theoretical solutions have been obtained on the basis of certain reasonable simplifying assumptions. In these solutions, the contributions of the unsteady boundary layer activities, the unsteady wake, and the trajectory of the passing vortex to the unsteady aerodynamic load are expressed explicitly as separate terms (Reference 12). The relative importance of each contributors is therefore easily assessed. During the reporting period, this relative importance has been systematically evaluated. The dependency of the aerodynamic load on various geometric and motion parameters has also been determined. These parameters include the freestream velocity, the strength and the initial position of the vortex, the angle of attack and the shape, including the camber and the thickness of the airfoil. The theoretical study indicates that large unsteady aerodynamic forces are possible even when the airfoil-vortex encounter does not induce massive flow separation. This type of encounter, designated an attached encounter, is obviously of importance in engineering.

(2) The simplifying assumptions used in the theoretical study have been removed by the use of the simplified zonal analysis. It has been demonstrated conclusively that the simplified zonal analysis requires very little computer time. Furthermore, the simplified zonal analysis produces highly accurate results for attached encounters. The simplified zonal method is therefore ideally-suited for parametric studies needed in design applications.

Figure 1 shows two unsteady lift curves for a vortex-airfoil encounter problem. The dashed curve is obtained using the simplified zonal method. The solid curve is obtained using the full Navier-Stokes zonal method. The results are for the case of an NACA 0012 airfoil at 0° angle of attach. The strength of the passing vortex is 0.2 times the product of the freestream velocity and the chord length. The initial position of the passing vortex is 0.26 chord below the centerline of the airfoil. The excellent quantitative agreement between the two sets of result clearly demonstrates the effectiveness of the simplified zonal method in predicting unsteady aerodynamics involving attached encounters. Particularly encouraging is the fact that the details of the lift curve, including the notch near the vortex position $x_\Omega/c = 1.0$ (x_Ω is the stremwise distance of the vortex from the airfoil leading edge and c is the chord length) is accurately predicted by the simplified zonal method. This behavior of the lift curve is discussed in connection with the full Navier-Stokes zonal solution below. It needs to be pointed out that the Navier-Stokes solution shown in Figure 1 has been thoroughly calibrated and is a useful standard against which simplified zonal solutions can be measured. The agreement between the simplified solution and the Navier-Stokes zonal solution exhibited in Figure 1 is typical for all problems studied under the present program.

(3) The Navier-Stokes results shown in Figure 1 is for a turbulent flow at a Reynolds number of 10^6 based on the freestream velocity and the chord length. The Baldwin-Lomax two-layer algebraic model is used in the solution procedure. The case shown in Figure 1 is an attached encounter case. Many cases involving detached encounters have been studied and reported in Reference 11. The details of the computation procedure is also reported in

Reference 11. The Navier-Stokes results shown in Figure 1 have been compared with appropriate computational results obtained by other investigators using different numerical methods to solve the Navier-Stokes equations. Excellent agreements have been observed. The general shpae of the lift curves shown in Figure 1 have been correctly predicted through the theory of viscous aerodynamics. The notch in the lift curve occurring near $x_{\Omega}/c = 1.0$ is due to a change of the sense of the wake vorticity being shed. This change occurs as the passing vortex flows by the airfoil's trailing edge. In many Navier-Stokes solutions obtained by other researchers, this notch, which must be present physically, is not predicted.

b. <u>Rapidly Pitched Airfoil</u>

I. Tuncer has recently completed an extensive investigation of the rapidly pitched airfoil problem using the theoretical, the simplified zonal and the full Navier-Stokes zonal methods. The results obtained have been comapred with experimental data that became avilable in the open literature only relatively recently (References 13 to 15). Conclusions derived from these studies regarding the range of applicability, the efficiency and the accuracy of each of the three methods are similar to those described earlier for the vortex-airfoil encounter problem. In the following, sample results obtained using the simplifed and the full Navier-Stokes zonal methods are presented and compared with available experimental data.

Figure 2 shows two time-dependent lift curves for a rapidly pitched NACA 0012 airfoil obtained respectively through the simplified zonal analysis and the full Navier-Stokes method. These two curves are compared with the experimental data of Reference 13. The airfoil, initially at 0° angle of incidence (α) in a uniform freestream, is pitched at a constant rate to a maximum 34.4° incidence angle. The airfoil is then held at this maximum incidence angle. The constant pitch rate is 0.158 times the freestream velocity divided by the chord. The flow Reynolds number is 80,000. The Navier-Stokes results shown in Figure 2 are obtained assuming the flow to be laminar.

Figure 2 shows that the results of the full Navier-Stokes zonal analysis are in better agreement with the experimental data than are the results of the simplified zonal analysis. In particular, the lift slope as predicted by the Navier-Stokes solution is in excellent agreement with the experimental lift slope for incidence angles up to about 20°. This incidence angle surpasses substantially the static stall angle. For incidence angles greater than about 20°, there exist differences between the results obtained in the present study and the experimental data. Similar observations can be made regarding the drag coefficient, as discussed in References 16 and 17.

Figure 3 shows a sequence of flow patterns obtained using the Navier-Stokes zonal solver. The lines shown around the airfoil are instantaneous streamlines in a reference frame attached to and rotating with the airfoil. These stremalines are plotted for stream function values with 0.05 increments. These flow patterns show that trailing-edge flow separation initiates at a very low incidence angle. As the airfoil is pitched to higher incidence angles, the trailing-edge separation region expands upstream continually. The formation of a leading edge vortex occurs as the incidence angle reaches 17°. As the pitching motion continues, the trailing edge and the leading edge vortices grow in size and eventually merge together. The merged vortex eventually bursts and moves away from the airfoil after the maximum incidence angle is reached.

Francis et al. (Reference 13) identified four typical shapes of pressure distributions on the surface of the rapidly pitched airfoil. They suggested that these shapes are related to four stages of development of a strong dynamic stall vortex. In stage (a), the dynamic stall vortex is not yet formed and the vortical region of the flow contains in essence only the attached zone. In stage (b), the vortex is formed and energized. In stage (c) the vortex loses energy while it is transported to the blocked "wake region" created by the large incidence angle of the airfoil. In stage (d), the flowfield relaxes to a state which is essentially one about a bluff body. Walker, Helin and their co-workers (Reference 14) subsequently presented flow visualization data showing the forming of strong leading-edge vortices. The streamlines shown in Figure 3 clearly show the forming, development and movement of a strong leading edge vortex during stages (b), (c), and (d), as surmised in Reference 13. In stage (a), a significant degree of trailing edge separation is observed. The presence of this trailing edge separation, however, does not alter the general shape of the pressure distribution. That is, the shape of the pressure distribution during stage (a) is similar to those observed for steady unstalled flows. In addition to trailing edge separation, the computed results show the presence of secondary vortices which, because of the large spacing between pressure measurement stations on the airfoil, are not observable in the experiments of Reference 13. The presence of the secondary vortices, however, has been noted by Robinson, Helin et al. (Reference 15). The computer time requirements of the new methods are very small. The Navier-Stokes results shown in Figures 2 and 3 were obtained using about 10 minutes of CYBER 990 CPU time. The simplified zonal method, in comparison, requires less than one minute of CYBER 990 CPU time.

Prior to the present reporting period, laminar flow results for rapidly pitched airfoils were obtained using the Navier-Stokes zonal method. These results indicate several issues needing additional attention. Among these issues is the question of flow instability and possible transition in the Reynolds number range of the experiments. The substantial trailing-edge separation observed in the computed result is expected to be suppressed by possible transition in the boundary layer. During the reporting period, the Navier-Stokes zonal solver has been further developed and refined. Several turbulence models have been studied. A two-layer algebraic model has been incorporated into the solution procedure. The fully established Navier-Stokes zonal solver has been used successfully in the investigation of turbulent flows around the rapidly pitched airfoil. The results of these investigations are included in I. Tuncer's Ph.D. thesis (Reference 17), scheduled for completion in the summer of this year.

c. Oscillating Airfoil

The fully established Navier-Stokes zones solver as well as the simplified zonal solver have been used in the computation of a variety of unsteady flow of practical interest. Under a separate research program, the problem of unstalled and dynamically stalled oscillating airfoils has been studied extensively during the reporting period. Because of the great importance of the problem in helicopter applications, definitive experimental data covering a broad range of circumstances have been obtained by various researchers in recent years. These experimental data have been used to calibrate the unified methods. In Figures 4, 5, and 6, selected results obtained using the new methods are compared with experimental data. These figures are included in the present report to demonstrate the high degree of accuracy achievable by the new methods. The Navier-Stokes computations are carried-out for turbulent flows, using the Baldwin-Lomax model.

In Figure 4 are shown the lift and the drag hystereses associated with a NACA-0012 airfoil undergoing sinusoidal oscillation in pitch between the incidence angles 0° and 20°, with a reduced frequency (k) of 0.25. For this case, although the maximum incidence angle is well above the static stall angle, the reduced frequency is sufficiently high to prevent dynamic stall. The lift loop obtained using the simplified zonal solver is in remarkably good agreement with the experimental data (Reference 18). This agreement is, in fact, better than that between the results of the full Navier-Stokes solver and the experimental data. The drag loop computed using the simplified zonal method, however, differs significantly from the experimental data. In contrast, the full Navier-Stokes drag results are in remarkably good agreement with the experiments.

In Figures 5 and 6 are shown results obtained using the full Navier-Stokes zonal solver for the NACA-0012 airfoil oscillating betweeing 5° and 25° compared with available experimental data (Reference 18). The flow Reynolds number is 10^6. The reduced frequency for the cases shown in Figures 5 and 6 are respectively 0.10 and 0.25. Figure 5 shows that, during the upstroke, the lift slope is nearly constant up to the incidence angle of 22°. Computed streamlines, given in Reference 17, indicate that a thin layer of reversed flow begins to appear near the trailing edge as the incidence angle reaches 12°. As the incidence angle increases, this thin layer of reversed flow extends further upstream towards the leading edge. However, the attached flow character is preserved well beyond the static stall angle. Not until the angle of 22° is exceeded when the abrupt separation of the boundary layer and the development of the leading edge vortex is observed. The leading edge vortex then grows, and eventually covers all of the upper surface. The formation and growth of this separation vortex is accompanied by a more rapid rise in the lift. This rapid rise in lift is followed by a period when the lift stays almost constant at its apex. When the incidence angle reaches 24°, the separation vortex bursts and sheds its vorticity into the wake. This process is manifested by the drastic decrease of lift near the end of the upstroke. This decrease is followed by a sharp rise in lift at the beginning of the downstroke. This rise in the lift is associated with the forming and the growth of a trailing edge separation vortex. After the incidence angle returns to 24.2°, the trailing edge vortex bursts and causes a sharp drop in the lift. This sharp drop is followed by the reattachment of the flow at the trailing edge and the graduate development of a secondary vortex structure. During this period, the lift drops gradually and eventually reaches a minimum value at an incidence angle of 11°, when the flow becomes fully attached again.

The behavior of the drag during the oscillation cycle is also shown in Figure 5. An interesting feature observed in Figure 5 is the spikes in the lift and the drag hysteresis loops that appear in the beginning stage of the downstroke. The appearance of the spikes is due to the formation of the trailing edge vortex immediately after the bursting of the strong separation vortex.

In Figure 6, the lift curve is shown for a higher reduced frequency case (k=0.25). The spike observed in Figure 5 is absent in Figure 6. The higher reduced frquency in this case shortens the time period for the upstroke. As a result, the formation of the leading edge separation vortex did not begin until the incidence angle reached 24.6° during the upstroke. The growth of this separation vortex, accompanied by a rapid rise in the lift, continues into the beginning phase of the downstroke. The bursting of the separation vortex and the drastic decrease of the lift occur at an incidence angle of 23.2° during the downstroke. This sequence of events produces a small loop in the maximum lift region in Figure 6 instead of the spike seen in Figure 5.

In Figure 7, pressure coefficients on the upper surface of the oscillating airfoil at a reduced frequency of 0.25 computed using the Navier-Stokes solver are compared with experimental data (Reference 18) at various angles of incidence during the oscillation cycle. Good agreement is observed. Additional results are presented in a future paper (Reference 19). Excellent agreement between the present results and available experimental data has been observed over a broad range of circumstances. The selected results presented here clearly show that all the details of the unsteady flow are correctly predicted. Quantitative differences do exist, but are in general within the uncertain bounds of the experiments. The good accuracy of the methods and the small computer time requirements make the methods developed under the present program ideally suited for unsteady aerodynamic analysis involving large forces.

3.2 Non-Rigid Two-Dimensional Bodies

The pervasive use of non-rigid lifting bodies by aquatic and aerial animals to generate large unsteady aerodynamic forces for propulsion and for control are well-known in the aerodynamics community. The superior ability of such animals in takeoff, landing and in-flight maneuver, though well-recognized, has not been successfully imitated in aeronautical engineering. It is clear that a prerequisite for achieving "supermaneuverability" through the use of large unsteady aerodynamic forces is the thorough understanding of the physical processes involved in the generation of unsteady forces by non-rigid bodies. In addition, accurate and effective predictive methods associated with non-rigid bodies are needed.

Unlike in the case of rigid bodies, definitive experimental data obtained under rigidly controlled environments are scarce in the case of non-rigid bodies. There exists, however, a reasonable amount of data acquired through zoological observations. In the present research program, several non-rigid body problems simulating well-known biofluiddynamic motions are selected for extensive theoretical/computational investigation. These selections are motivated by the availability of zoological data which are useful in guiding the research.

Two categories of non-rigid bodies -- the articulate and the flexible -- have been studied during the reporting period. It has been demonstrated conclusively that the conceptual framework of the unified theoretical/computational methods, previously established for the study of rigid lifting bodies, can be adopted for studies of non-rigid bodies. Substantial efforts have been devoted to the development of efficient and accurate procedures for non-rigid body problems during the reporting period. Unified methods have been used in the study of three non-rigid body problems. Selected results are described below to demonstrate the application of these new methods. The results indicate that very large aerodynamic forces are obtainable through unsteady motions of non-rigid bodies. Several issues which presented conceptual difficulties in the past have been successfully resolved during the reporting period. A particularly encouraging outcome of the present research is that quantitative as well as qualitative predictions of the present research, obtained in the absence of experimental data, have been substantiated recently by experiments conducted elsewhere (Reference 20). This role reversal -- instead of calibrating predictive methods using available experimental data, new experimental data are measured against available theoretical/computational predictions -- further confirms the state of maturity of the new methods.

a. Weis-Fogh Problem

The Weis-Fogh problem involves two identical wings moving relative to one another. The two wings undergo a motion cycle containing the fling, the separation, the return and the clap phases. During the fling phase of the motion, the two wings, with their trailing edges joined together, have their leading edges flinging apart. This fling phase is followed by the separation phase during which the wings move in opposite directions and behave as conventional wings. After reaching certain separation distance, the wings flip and approach each other during the return phase. The wings then rotate about their leading edges until their trailing edges are joined together. The Weis-Fogh problem is of biofluiddynamic interest. The motion cycle just described represents the wing beat of an insect (Encarsia Formosa) in hover. In the present research, the aeronautical engineering implications of this motion are investigated.

Theoretical studies of the fling phase of the Weis-Fogh motion were carried out prior to the present reporting period using the viscous theory of aerodynamics. In these studies, the two wings were represented by flat plates in two dimensions. These theoretical results are documented in an earlier Ph.D. thesis prepared by H. Hu-Chen (Reference 21).

During the reporting period, studies have been made of the Weis-Fogh wings undergoing the fling and the separation phases of the motion cycle. Both the full Navier-Stokes and the simplified zonal methods have been used in these studies. The results are documented in the Ph.D. thesis of M. Sohn (Reference 8) and of R.F. Reilman (Reference 22), the latter is being finalized at the present. The Weis-Fogh problem is a well-known biofluiddynamics problem and is concerned with the hover of insects, as previously stated. The wing motion considered in all previous studies takes place in a stationary air. In engineering applications, time-averaged motion of the wing pair relative to the freestream is of interest. In the work of Reilman (Reference 22), who developed and utilized a simplified zonal method, effects of the mean motion of the wings relative to the freestream along a plane of symmetry is studied.

Although a number of distinguished aerodynamicists, including Lighthill (Reference 23), studied the Weis-Fogh problem in the past, no theoretical or computational prediction of the unsteady aerodynamic load experienced by the wings was obtained. Indeed, it is well-known that previous methods of aerodynamic analysis are not suitable for the study of complex unsteady flows such as the ones associated with the Weis-Fogh problem. The results obtained by Hu-Chen (Reference 21) using the new theoretical method developed under the present program represents the first successful solution of the Weis-Fogh "aerodynamics" problem.

Maxworthy carried out flow visualization experiments and presented streakline photographs for the fling phase (Reference 24). No experimentally measured unsteady aerodynamic forces, however, were available in the literature at the time Hu-Chen completed her theoretical studies. Hu-Chen obtained closed-form analytical expressions for the lift, the drag, and the power expenditure of the Weis-Fogh wings during the fling phase. With these expressions, not only the aerodynamic performance of the Weis-Fogh wings are easily evaluated, but also the major contributors of the performance are identified and their relative importance determined. Detailed discussions of the effects of the geometric and motion parameters (opening angle, velocity, and acceleration of the wings, etc.) are presented along with the closed-form formulas in Reference 21. In the absence of existing experimental or analytical results, it was not possible at the time Hu-Chen completed her studies to verify the accuracy of the theoretical results. Using the closed-form solution, however, Hu-Chen determined the unsteady lift force that should have been experienced in a specific experiment carried out by Maxworthy. This theoretical prediction has been confirmed by more recent experiments of Spedding and Maxworthy. Figure 8, presented by Spedding and Maxworthy in Reference 20, compares their experimentally determined lift curve with the theoretical result obtained using the closed-form solution. In view of the fact that the theoretical solution was developed prior to the conduct of the experiment, the good agreement between the theoretical prediction and the experimental information is especially encouraging. This good agreement confirms the reliability of the new methods.

In Figure 9 are shown two sets of streamline patterns around a pair of Weis-Fogh wings obtained using, respectively, the simplified zonal method (Reference 22) and the Navier-Stokes method (Reference 8). These streamline patterns are compared with a streak photograph, also shown in Figure 9, presented by Maxworthy (Reference 24) for the same geometric and motion parameters. The good agreement between the analytical and the experimental results again attests to the validity of the simplified zonal approach in treating complex unsteady aerodynamic problems.

b. Oscillating Posterior Plate Problem

J. Singh (Reference 10) recently completed a study of a problem of an articulate body with an oscillating posterior plate using the simplified zonal method. The articulate body is formed by joining together an anterior plate and a posterior plate. This articulate body moves relative to the freestream. The anterior plate moves at a constant velocity with a zero incidence angle. The leading edge of the posterior plate is joined to the trailing edge of the anterior plate. In addition to translating with the anterior plate, the posterior plate oscillates in pitch sinusoidally about its leading edge. This problem simulates the swimming motion of an aquatic animal such as a fish or a dolphin for which a substantial amount of zoological information is available. In aeronautical engineering, this problem represents the large-amplitude time-dependent motion of a large control surface relative to a wing in forward flight.

Reference 5 reviews the simplified zonal method for two-dimensional unsteady aerodynamics and presents selected results for the oscillating posterior plate problem. As discussed earlier, the simplified zonal method is a unified theoretical-computational method. Using the viscous theory of aerodynamics (Reference 4) and the integral representation of the kinematics of viscous flow (Reference 6), closed-form analytical expressions have been obtained for the strength (γ) of the vortex sheet approximating the unsteady boundary layers surrounding a lifting body. Using these analytical expressions, closed-form solutions have been obtained for the unsteady aerodynamic force experienced by the lifting body. This simplified zonal method is applicable to various articulate and flexible lifting bodies. In theoretical solutions, the contributions of the various motion and flow parameters to the unsteady aerodynamic force are present as distinctive terms. As a result, each contribution can be evaluated individually and their relative importance assessed.

Closed formed solution for the oscillating posterior plate problem has been obtained during the reporting period. This solution (Reference 5) contains the contributions of the freestream velocity, the rotation of the two plates together as a rigid body, the rotation of the two plates relative to each other, and of the wake. The following equation summarizes the theoretical solution.

$$C_F = C_{F_1} + C_{F_2} + C_{F_3} + C_{F_4} \tag{7}$$

where C_F is the unsteady aerodynamic force coefficient, with the dynamic head times the total length of the two plates as the reference force, and C_{F_1}, C_{F_2}, C_{F_3}, and C_{F_4} are respectively the contributions of the four contributors just stated.

The component force coefficients are expressed below using notations of complex variable. In these expressions, the real parts represent the drag and the imaginary parts represent the lift.

$$C_{F_1} = \frac{\Omega c}{U_\infty} \frac{d}{d\beta} \{ g(\beta)[1+2(\frac{\beta}{\pi})^2 e^{i\beta}-e^{i\beta}] \} \tag{8}$$

$$C_{F_2} = \frac{c^2}{U_\infty^2} \frac{d}{dt} \{ \Omega h(\beta)[\frac{1}{4}-(\frac{\beta}{\pi})^2] \frac{e^{i\beta/2}}{\cos\beta} \} \tag{9}$$

$$C_{F_3} = \frac{c^2}{U_\infty^2} \frac{d}{dt} \{ i\Omega h(\beta)[1-(\frac{\beta}{\pi})^2] \frac{e^{i\beta/2}}{\sin\beta} \} \tag{10}$$

where

$$g(\beta) = \frac{\pi}{2} (1+\frac{\beta}{\pi})^{-(1+\frac{\beta}{\pi})} (1-\frac{\beta}{\pi})^{-(1-\frac{\beta}{\pi})} \tag{11}$$

$$h(\beta) = \frac{2}{3} (1+\frac{\beta}{\pi})^{-\frac{3}{2}(1+\frac{\beta}{\pi})} (1-\frac{\beta}{\pi})^{-\frac{3}{2}(1-\frac{\beta}{\pi})} \beta \tag{12}$$

and

$$C_{F_4} = \frac{i}{U_\infty^2 c} \sum_k \Gamma_k \frac{d}{dt} [ke^{-i\alpha} (\frac{1}{2\zeta_k} - \frac{\bar{\zeta}_k}{2}) - \overline{f(\zeta_k)}] \tag{13}$$

In Eq. (13), z_k is the location of the kth wake vortex in the physical plane and ζ_k is the corresponding location in a conformally transformed plane.

It is worthy of note that the closed-form solutions clearly express the influences of the freestream velocity U_∞, the chord length c, the angular velocity Ω, the angular acceleration $\frac{d\Omega}{dt}$, and the incidence angle β of the posterior plate. For example, from Equations (9) and (10), it is easy to see that the unsteady force associated with the rotation of the plate is directly proportional to the square of the reference time (c/U_∞) during which the articulate body moves a chord length relative to the freestream. The dependency of this force on β, Ω, and $d\Omega/dt$ are given directly by the terms inside the brackets in Equations (9) and (10).

The above solution has been used to evaluate quantitatively the unsteady aerodynamic force acting on the articulate body under a broad range of circumstances. It has been shown that the results are in general agreement with zoological observations. For low amplitude motion, linearized aerodynamic analyses yield results in agreement with the present solution. It should be emphasized that the merit of the new methods developed under the present research program lie mainly in the non-linear regime where previous methods are inadequate. The solutions obtained, discussed in detail in References 10 and 21, indicate that exceedingly large unsteady aerodynamic forces are obtainable using articulate bodies undergoing large amplitude motion.

c. Flexible Body Problem

During the reporting period, procedures for the aerodynamic analysis of flexible bodies undergoing time-dependent motion have been developed using the simplified zonal method. Singh (Reference 10) computed the strength of γ of the vortex sheet approximating the vorticity of the unsteady boundary layers using a vortex panel method. The strength γ is then placed into Equation (5) to evaluate the unsteady aerodynamic force. This procedure is quite efficient and is useful in a variety of circumstances involving flexible bodies.

Detailed descriptions of the solution procedure developed by Singh is presented in his doctoral thesis (Reference 10) scheduled for completion in the summer of 1988. Singh used a problem of 'steady swimming' of fishes as a test problem in his study. The term 'steady swimming' is used by zoologists to describe fish swimming at a constant mean velocity. In reality, a steadily swimming fish makes undulatory or oscillatory motion of its body to generated unsteady aerodynamic forces. The mean thrust generated by the fish counters the mean drag experienced. Steady swimming of live fish has been observed and extensively recorded. The bulk of the recorded data about fish swimming consists of parameters such as the amplitude of the tail motion, the tail-beat frequency and the length of the propulsive wave. Not much is known about the detailed time-dependent motion of the fish in the steady swimming mode. A generally accepted model for steady swimming views the fish body as composed of a series of small segments. Each segment undergoes a periodical transverse motion. The amplitude of this transverse motion is small for the anterior part of the fish. The amplitude increases towards the tail of the fish and reaches a maximum at the tail's trailing edge. This model is compatible with the concept of the vortex panel method.

Singh used a marker particle procedure to obtain wake flow patterns behind a steadily swimming fish. Figure 10 shows such a pattern for a fish, represented by a 9% thick flexible airfoil. The reduced frequency, $\sigma = \Omega c/U_\infty$, where Ω is the angular velocity of the periodic motion of the fish, for the case shown in Figure 10 is 4. Also shown in Figure 10 is a wake pattern drawn from a flow visualization study of a live fish (Reference 25). The qualitative agreement between the observed pattern and the present computed pattern is quite good.

A quantitative calibration of Singh's procedure is shown in Figure 11. Kelly et al. (Reference 26) carried out experiments and obtained time-averaged thrust coefficients using two different two-dimensional flexible hydrofoils, one made of rubber, and the other made of metal. Figure 11 shows a comparison between the present results and Kelly's data over a broad range of reduced frequency. This figure shows that, for higher reduced frequency values, the present results lie between Kelly's data for the two hydrofoils. For lower reduced frequency values, flow separation was observed experimentally. This fact is consistent with the observation that the measured thrust is lower than the present predicted result.

In addition to the work summarized above, substantial efforts have been devoted during the reporting period to the development of time-dependent curvilinear coordinate systems for flexible lifting bodies. The conceptual development of a method of unsteady aerodynamic analysis using time-dependent coordinates systems has been completed. With this method, a time-dependent curvilinear coordinate system is first constructed through a coordinate transformation. The flexible lifting surface, which undergoes unsteady motion, is mapped onto a stationary (time-independent) segment of a coordinate surface in the transformed space. Aerodynamic analysis is then carried out in the transformed space. With the lifting surface made a fixed part of a coordinate surface, procedures for aerodynamic analysis become substantially simpler.

Prior to the present reporting period, the method of time-dependent coordinate systems was explored through a theoretical study of the two-dimensional Weis-Fogh problem. Although the method was shown to be poserful, it was thought that only a severly restricted class of flexible lifting surfaces can be mapped onto a fixed segment of a transformed coordinate surface. For this reason, the method was believed to have a limited range of application. During the reporting period, however, a numerical procedure for constructing time-dependent curvilinear coordinates involving two spatial dimensions has been developed. This procedure is generally applicable to various flexible surface geometries. The restriction in the range of applications of the time-dependent coordinate system method in two-dimensional applications is successfully removed.

3.3 Three-Dimensional Lifting Bodies

A pilot computer code using selected concepts of the full Navier-Stokes zonal method has been prepared during the reporting period. This code has been used in the computation of flows around several wings. Selected results obtained using this pilot code are presented to illustrate the current stage of development of the three-dimensional methods.

Figure 12 shows a C-H grid system and a C-O grid system, both developed for the present study. The wing shown in Figure 12 is an NACA 0012 rectangular wing with an aspect ratio of 4. Similar grid systems around several other wings have been developed. In the present report, results obtained for the NACA 0012 wing and for a flat plate wing, also with an aspect ratio of 4, are briefly described. Flows studied are symmetric about the center planes of the wing span. In each figure presented here, only one-half of the flowfield is shown.

The present study has shown that the C-H grid is better suited for three-dimensional viscous flow computations. Efforts have been initiated to develop adaptive grid systems ideally suited for special flow features, such as the tip vorticity systems (tip vortices), that are present in three-dimensional viscous flows.

Figure 13 shows streamline patterns around the NACA 0012 at an angle of attack of 4° and around a flat plate wing at an angle of attack of 10°. The presence of the tip vortices is evident in Figure 13. Streamlines in the tip region of each wing wrap around the wing tip and then form bundles. The flow around the flat plate wing, because of its higher angle of attack, contains a more prominent tip vortex. The flows studied are low Reynolds number laminar flows.

Figure 14 shows constant vorticity contours in wake planes of the two wings. Figure 14(a) shows that, in the wake plane one chord length downstream of the trailing edge of the flat-plate wing, the tip vortex system contains a region of counterclockwise vorticity (viewed from the wing). The distribution of vorticity in this region is nearly axi-symmetric about a center point. To the left of this region is a secondary region of clockwise vorticity. The vorticity in these two regions together form the tip vortex system. The coexistence of the two vortices are also observed in 14(b) which gives the vorticity contours in the wake plane one chord length downstream of the NACA 0012 wing. The differences between flow patterns shown in Figures 14(a) and 14(b) are discussed later.

Tip vortices are in reality continuations of the vorticity in the wing's boundary layers. In Figure 15 is shown a set of vorticity lines, defined as paths tangential to the vector vorticity field in the three-dimensional flow, associated with the flat plate wing. Vorticity lines associated with the boundary layer on the upper surface of the wing are shown as solid lines and those associated with the boundary layer on the lower surface are shown as broken lines. The streamline patterns in Figure 13, together with the vorticity lines in Figure 15, indicate that the vorticity of the primary (counterclockwise) vortex originates from the upper boundary layer while the vorticity of the secondary (clockwise) vortex originates from the lower surface.

In Figure 16, the vorticity surface contour in the wake plane of the flat plate wing is shown. This figure clearly indicates that the total strength of the clockwise vortex, the one originating from the lower surface of the wing, is very small compared to that of the counterclockwise vortex. Vorticity contours for various wake planes at various wake locations, not shown here, indicate that the secondary (clockwise) vortex rotates counterclockwise around the primary vortex as it moves downstream in the tip region. This rotation results from the strong counterclockwise total vorticity in the primary vortex. The secondary vortex is eventually brought to the left of the primary vortex, as is shown in Figure 14(a). In Figure 14(b), the rotation of the secondary vortex is still in progress. Since the wing in this case is at an angle of attack of 4° (compared to 10° for the case shown in Figure 14[a]), the primary vortex is not as strong, and the secondary vortex has not yet been swept completely to the left of the primary vortex. Instead, the vorticity of the secondary vortex wraps around the primary vortex.

The existence of the counterclockwise vortex, which dominates the tip vortex system, is well-known. Indeed, this counterclockwise vortex is usually recognized as a part of the horseshoe vortex system in inviscid aerodynamic analyses. The present viscous flow analysis reveals the detailed structure of the tip vortex system. The existence of the secondary clockwise vortex and the significance of the detailed structure of the tip vortex system in unsteady aerodynamics were not widely recognized in previous studies of the tip vortex system. A literature survey carried out during the reporting period uncovered some earlier experimental data suggesting the presence of the secondary vortex in the tip region. A theoretical study using the viscous theory of aerodynamics has been initiated to assess the importance of the secondary vortex in unsteady aerodynamic applications. This study has confirmed the necessary presence of the secondary vortex as well as the necessary dominance of the primary vortex in the tip region.

Several issues relating to the analysis of unsteady aerodynamics involving three spatial dimensions are being investigated intensively. The first issue is concerned with the presence of the tip vortex region in three-dimensional computations. The tip vortex region, in fact, is a special flow zone which does not have a two-dimensional counterpart. This special flow zone possesses physical and mathematical characters absent in the two-dimensional flow zones. During the reporting period, a reasonable familiarity has been developed about the tip vortex system. Efforts have been initiated to fully exploit the advantages offered by the zonal methods in computing three-dimensional unsteady flows. These zonal methods, already fully established for two-dimensional problems, are expected to be equally powerful for three-dimensional analyses. In particular, special grid systems for the tip vortex region are being developed to provide a high degree of resolution, but not excessive computation demand, for this special zone. Approximations of the tip vortex zone using bundled free vortices are being examined. These approximations, like those established for two-dimensional simplified zonal analysis, do not involve an inviscid fluid assumption. They are being established on the basis of realistic approximations of the vortity distribution that actually exist in the tip vortex zone.

In the study of two-dimensional flows, the calculus of complex variables has been used to facilitate the theoretical analysis as well as the computation procedure. Since complex variables are not useful in studies of three-dimensional flow, new techniques need to be developed. In particular, in the kinematic part of the analysis, the generalized Biot-Savart law relating the velocity field to the vorticity distribution is not directly applicable in a transformed space. As a result, the extremely efficient procedures for computing the boundary vorticity values and the velocity values, fully developed for two-dimensional unified theoretical-computational analyses, need to be revised.

The unified theoretical-computational methods, because of their ability to confine the solution field to the viscous zones of the flow, have led to drastic (more than a factor of 10) reductions in the required amount of computation for two-dimensional flows. The achievable factor of reduction for three-dimensional flows is obviously greater since the three-dimensional solution field is reduced along two directions rather than along one direction for two-dimensional flows. It is important to note that, at the present, the solution of complex three-dimensional problems using conventional finite-difference and finite-element methods requires excessively large amounts of the amount of computation even on the most powerful supercomputers. The reduction of computation offered by the unified theoretical-computational methods is therefore critically important. During the reporting period, the practicality of using multi-processing ability of supercomputers in conjunction with the zonal concept has been studied. Procedures employing multi-processors to treat simultaneously the various viscous flow zones that coexist in a viscous unsteady three-dimensional flow have been conceptualized. Estimates of required operation counts have been made, and they show that, when suitably developed, zonal methods based on the present unified theoretical/computational approach will require less than 30 minutes of time on a supercomputer, such as a GRAY-XMP or a CRAY 2, operating in the multiprocessor mode. Current efforts are devoted to the detailed development of such a three-dimensional zonal procedure for multi-processing. During the reporting period, gratuitous computer time has been obtained on the GRAY XMP computer located at Pittsburgh Supercomputer Center (Grant #PSCA88) through the National Science Foundation and on the Numerical Aerodynamic Simulation (NAS) facility at the NASA - Ames Research Center. The possibility of longer-term arrangements is being investigated.

4. CONCLUDING REMARKS

During the reporting period, unified theoretical-computational methods for unsteady viscous aerodynamic analysis have been fully developed, documented and utilized in comprehensive studies of a broad range of two-dimensional problems. An extensive information base has been created using the unified methods. This information base has been synthesized and utilized to produce a reasonable understanding of the important physical processes contributing to the generation of large unsteady forces. Efforts are in progress to develop unified theoretical-computation methods for three-dimensional viscous unsteady aerodynamic analysis. Results have been obtained during the reporting period for pilot three-dimensional problems.

The present stage of development of three-dimensional predictive methods is comparable to that of two-dimensional predictive methods in 1985. At that time, all conceptual issues related to two-dimensional methods were identified. Results of some two-dimensional pilot problems were obtained. These two-dimensional methods, however, were not at that time sufficiently advanced to meet the stringent requirements and expectations of the present researchers. Based on the work already completed and the present rate of progress, the goals and milestones set forth in the proposal (Reference 2) for the present, final, phase of this research program is judged to be realistic. Specifically, it is anticipated that routine and accurate predictive methods for three-dimensional unsteady aerodynamics will be made available in approximately three years, as was originally projected.

The predictive methods that have been developed under the present research program utilizes several unconventional concepts conceived at Georgia Tech. These unconventional concepts are based mathematically on the full viscous (Navier-Stokes) equations. They lead, however, to procedures that represent drastic departures from prevailing computation or theoretical procedures for aerodynamic analysis. During the reporting period, substantial efforts have been made to fully document the new methods. Extensive external interactions with leading aerospace companies, government laboratories and universities interested in non-linear unsteady aerodynamics have been carried out. Strong interests have been shown by several research organizations in the unique analytical capabilities established under the present program. At the present, theoretical and computational methods developed under the present program are being used by several research groups in the aerospace industry, government laboratories, and universities. These activities are expected to continue and to expand during the next few years.

5. PUBLICATIONS

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Ph.D. Dissertations Under AFOSR Support

1. H. Hu-Chen, "A Study of the Aerodynamic Performance of Weis-Fogh Wing," March 1985.

2. T.M. Hsu, "A Study of a Vortex-Lifting Surface Interaction Problem," March 1986.

3. M.Y. Sohn, "A Numerical Study of the Weis-Fogh Mechanism," August 1986.

4. R. Reilman, "A Theoretical and Numerical Study of the Weis-Fogh Problem Including Wing-Separation," in preparation.

5. J. Singh, "A Computational Study of the Vortex Trapping Phenomena," in preparation.

6. I. Tuncer, "Theoretical and Computational Studies of Dynamic Stall of Rapidly Pitched Airfoil," in preparation.

7. M. Patterson, "Zonal Computation of Compressible Navier-Stokes Equations," in preparation.

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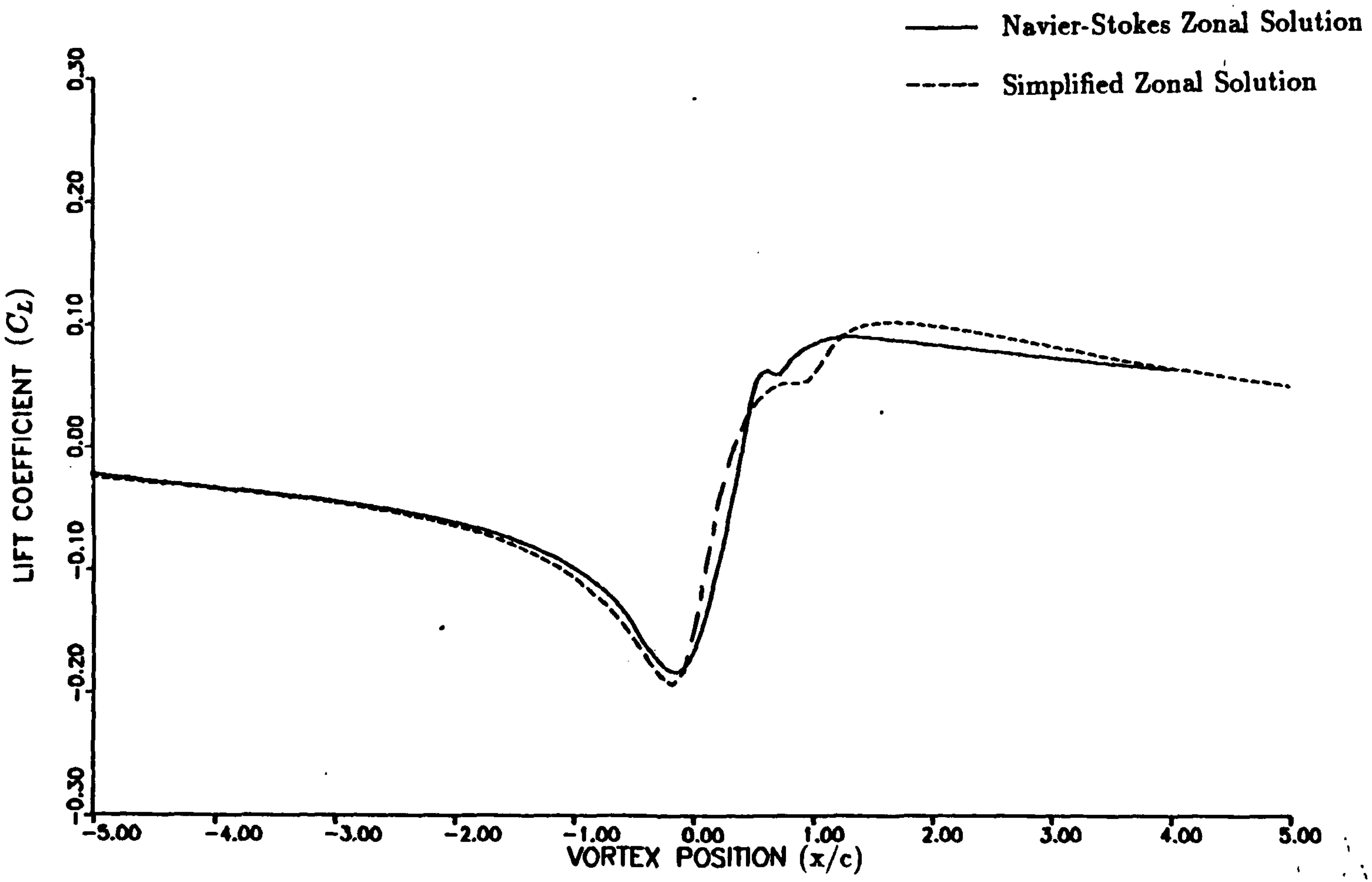


Figure 1. Unsteady Lift of Turbulent Attached Airfoil-Vortex Interaction.

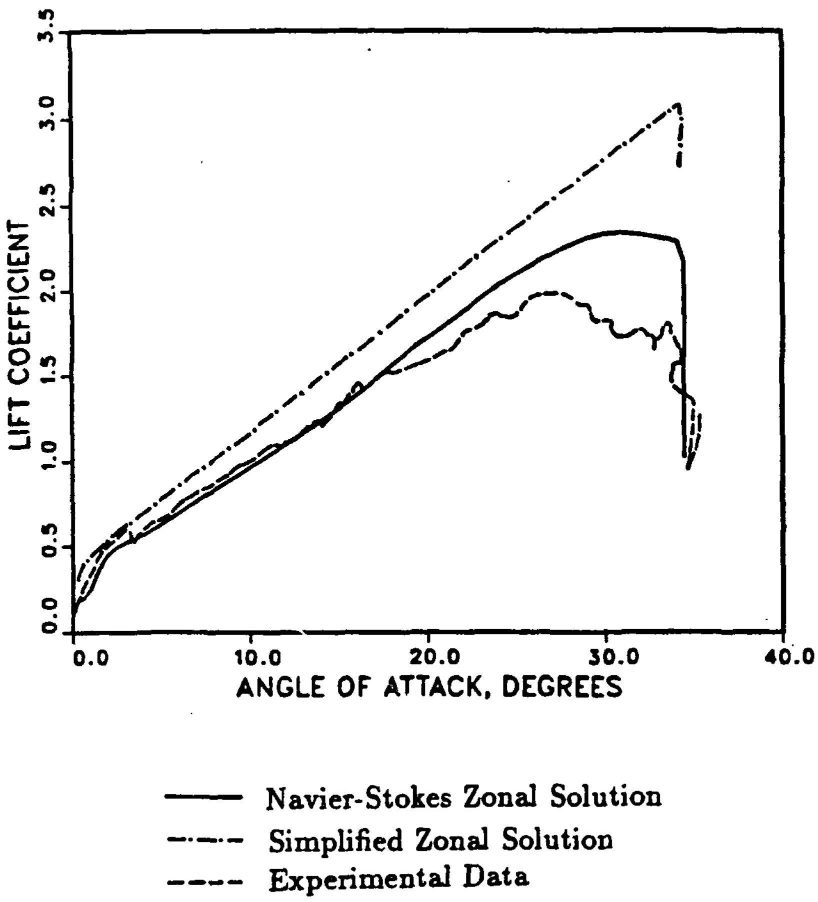


Figure 2. Lift Coefficient for a Rapidly Pitched NACA-0012 Airfoil.

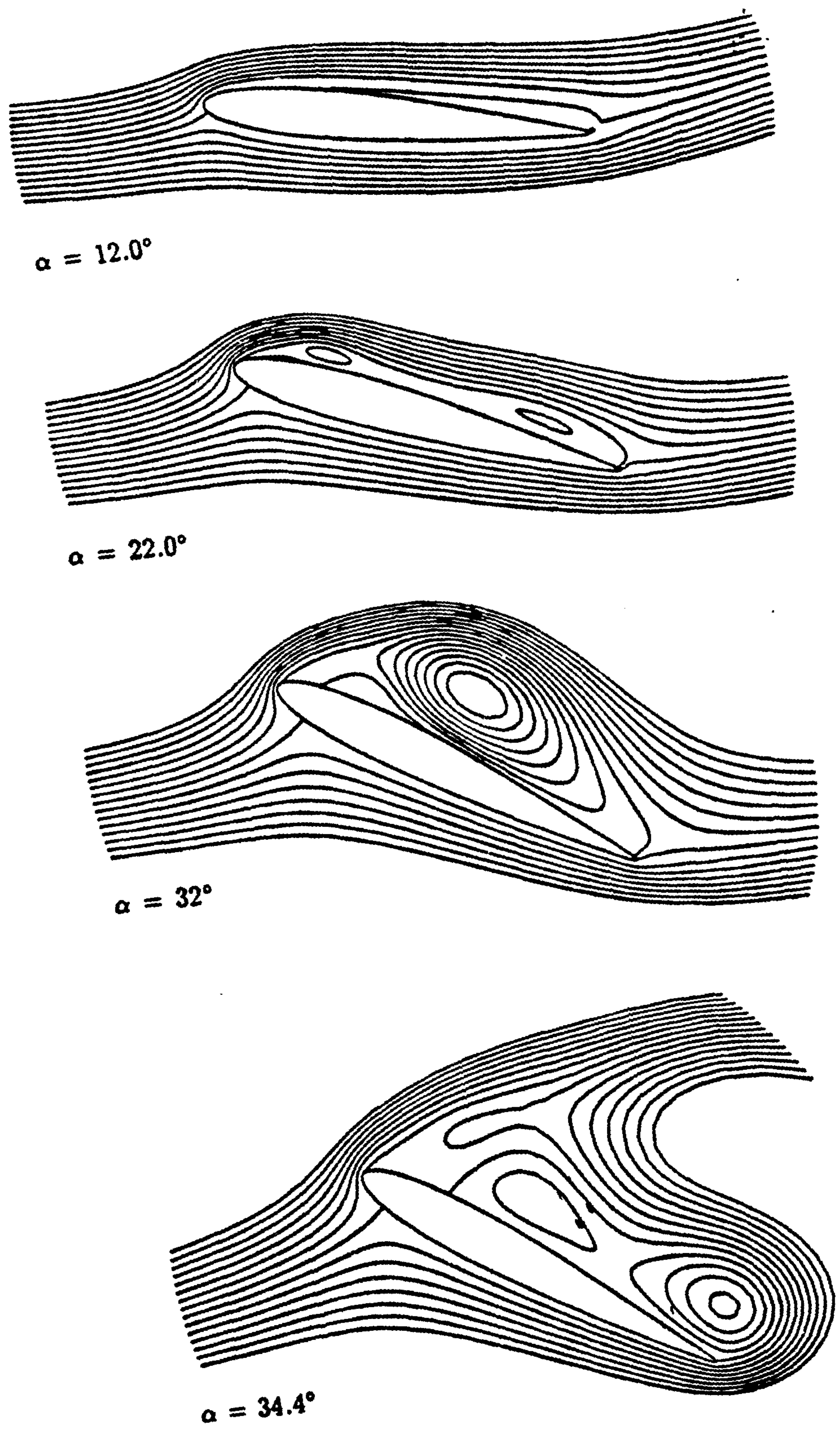
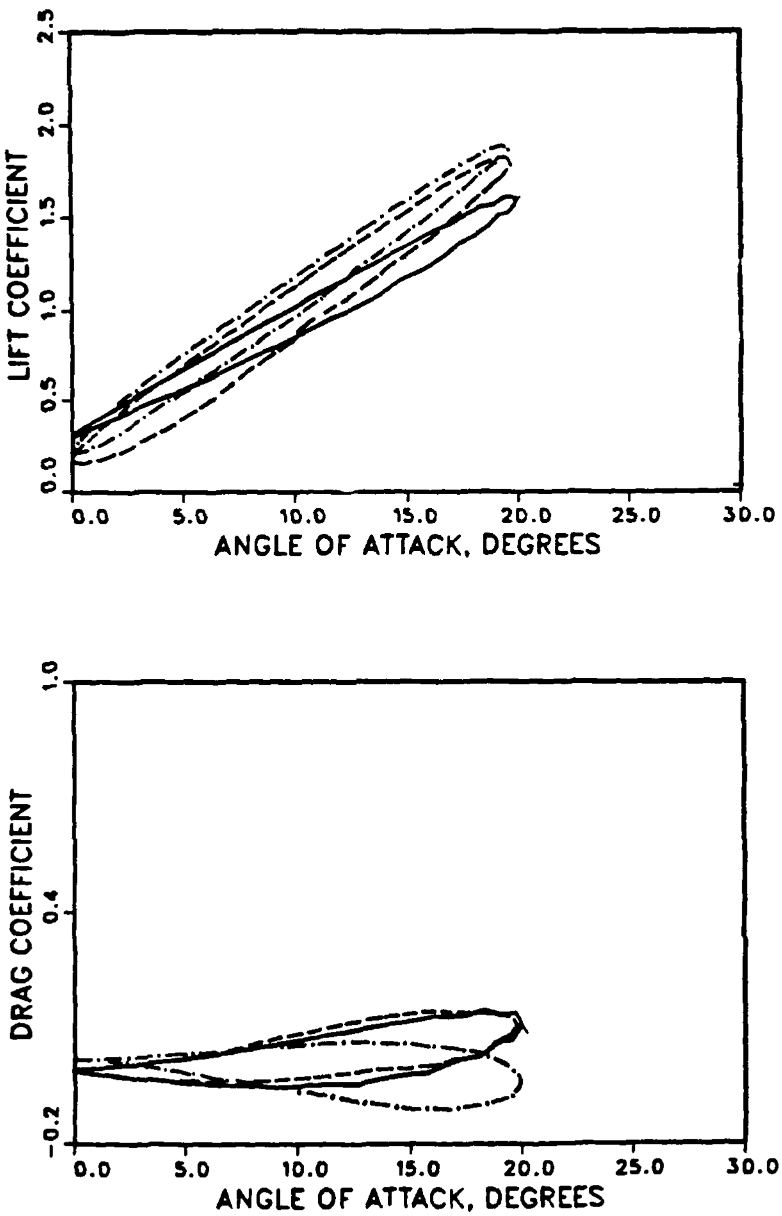


Figure 3. Streamlines around a Rapidly Pitched NACA-0012 Airfoil.



——— Navier-Stokes Zonal Solution
-·-·- Simplified Zonal Solution
- - - - Experimental Data

Figure 4. Lift and Drag Hysteresis loops of a Rapidly Oscillating NACA-0012 airfoil.

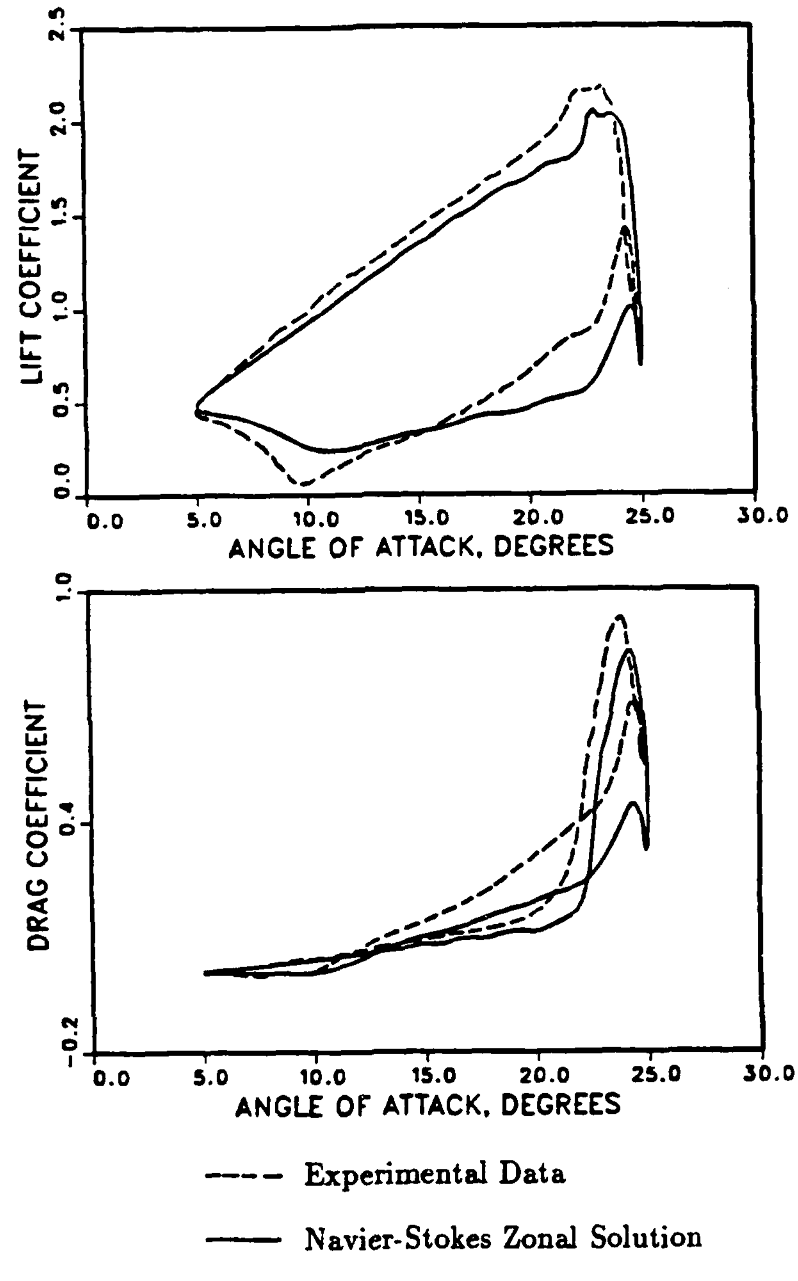
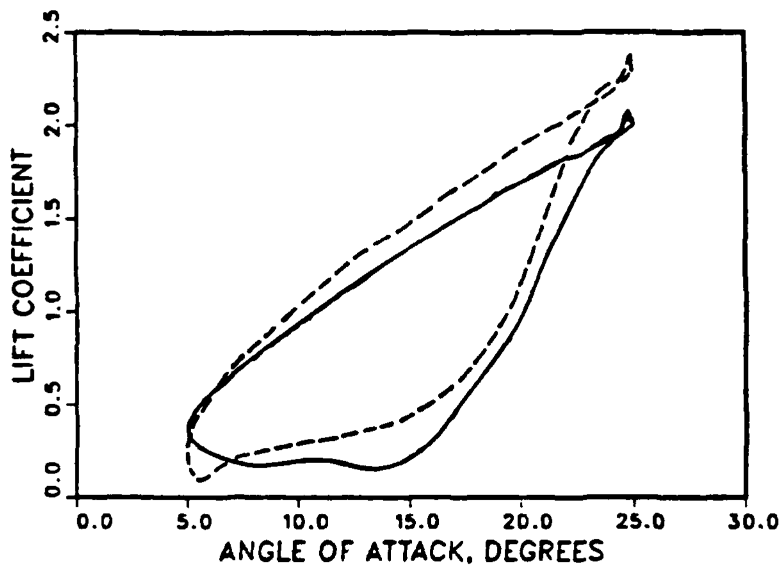


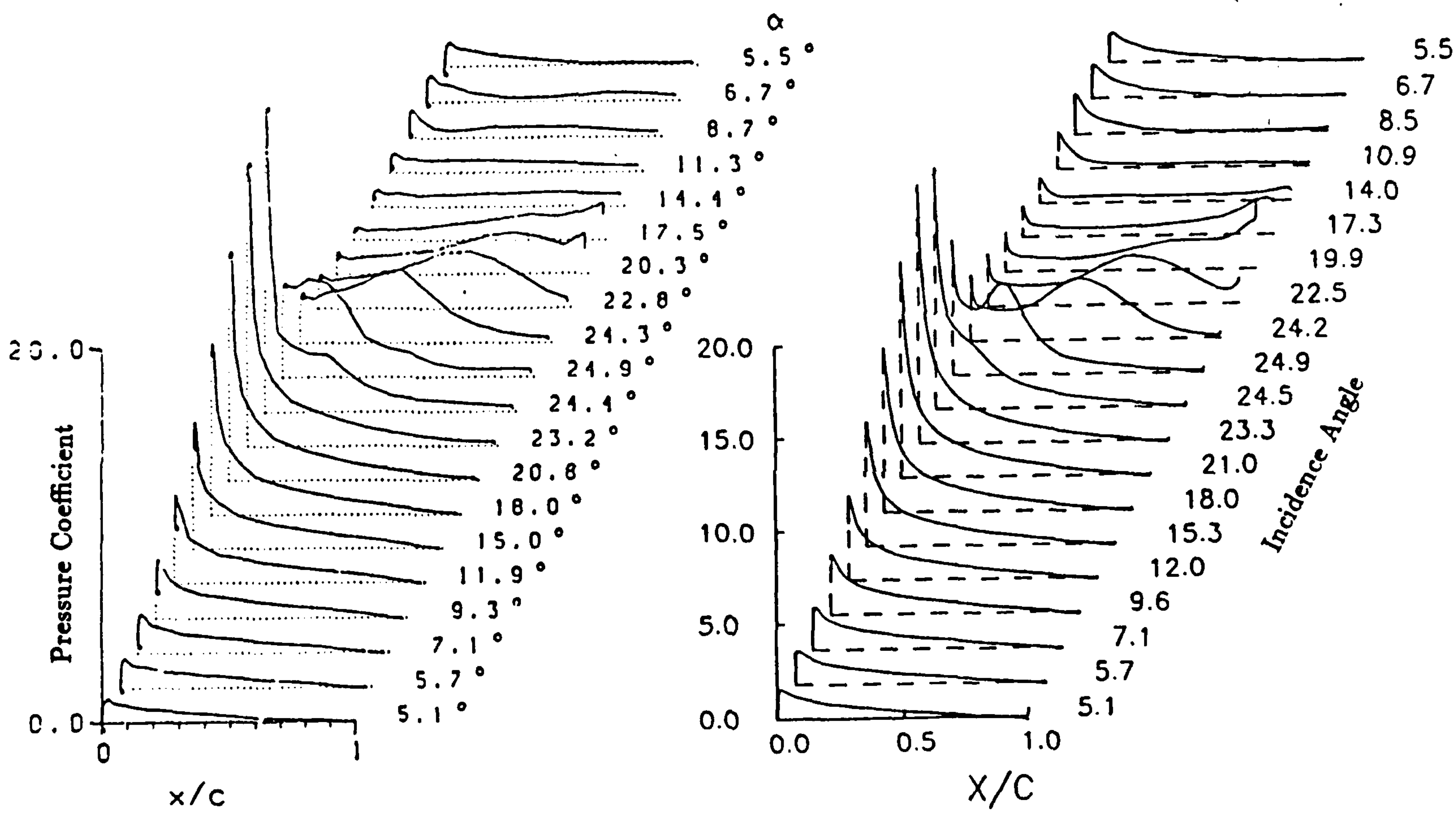
Figure 5. Dynamic Stall Hysteresis Loops of an NACA-0012 Airfoil Oscillating at a Reduced Frequency of 0.10



----- Experimental Data

——— Navier-Stokes Zonal Solution

Figure 6. Lift Hysteresis Loop of an Oscillating NACA-0012 Airfoil in Dynamic Stall at a Reduced Frequency of 0.25



(a) Experimental Data

(b) Navier-Stokes Zonal Solution

Figure 7. Pressure Distributions on the Upper Surface of an Oscillating NACA-0012 Airfoil in Dynamic Stall.

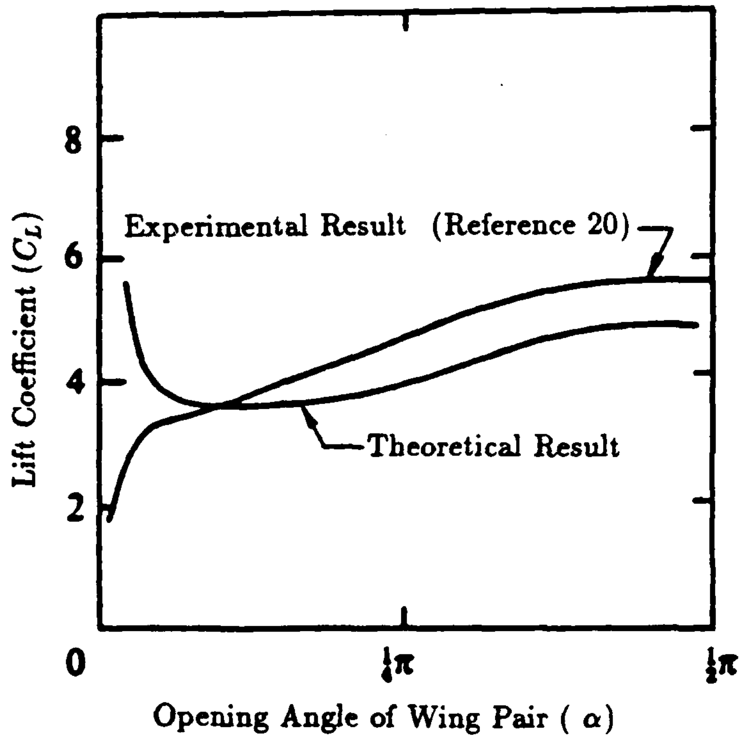
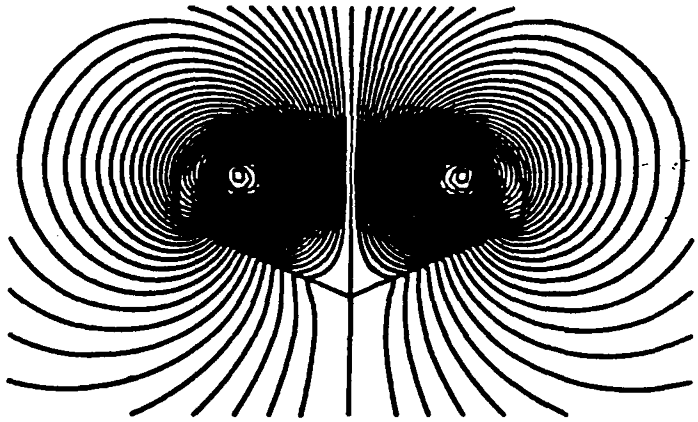
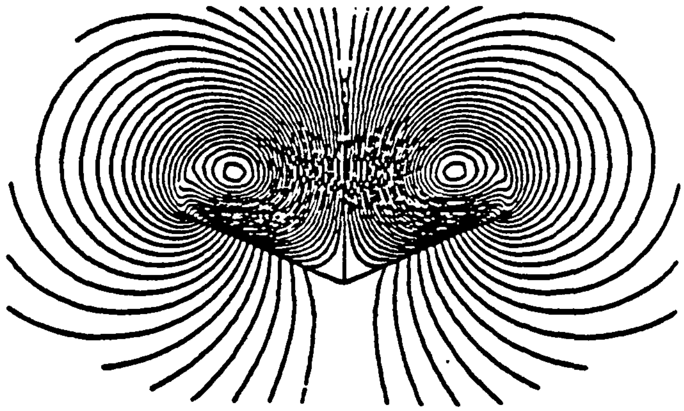


Figure 8. Lift Coefficient for Fling Phase of Weis-Fogh Wings.



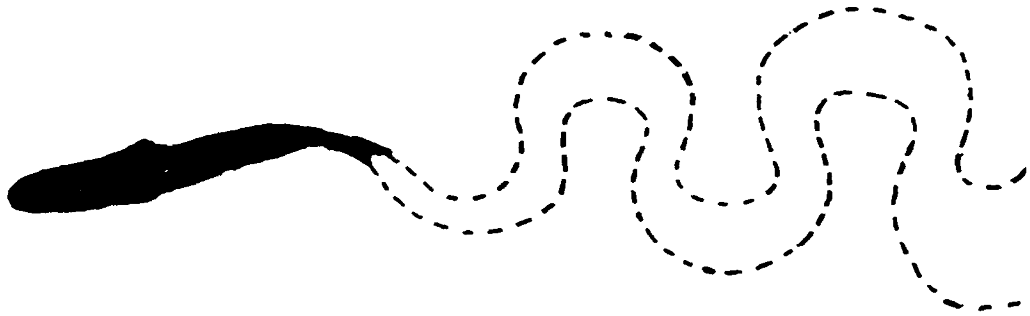
Simplified Zonal Solution



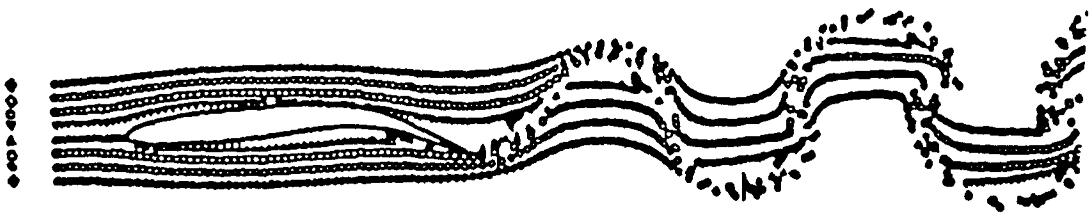
Full Naviers-Stokes Zonal Solution

Flow Visualization (Reference 24)

Figure 9. Flow Pattern associated with Weis-Fogh Wings in Fling.



Live Fish Observation (Reference 25)



Simplified Zonal Solution

Figure 10: Wake Behind a Steadily Swimming Fish

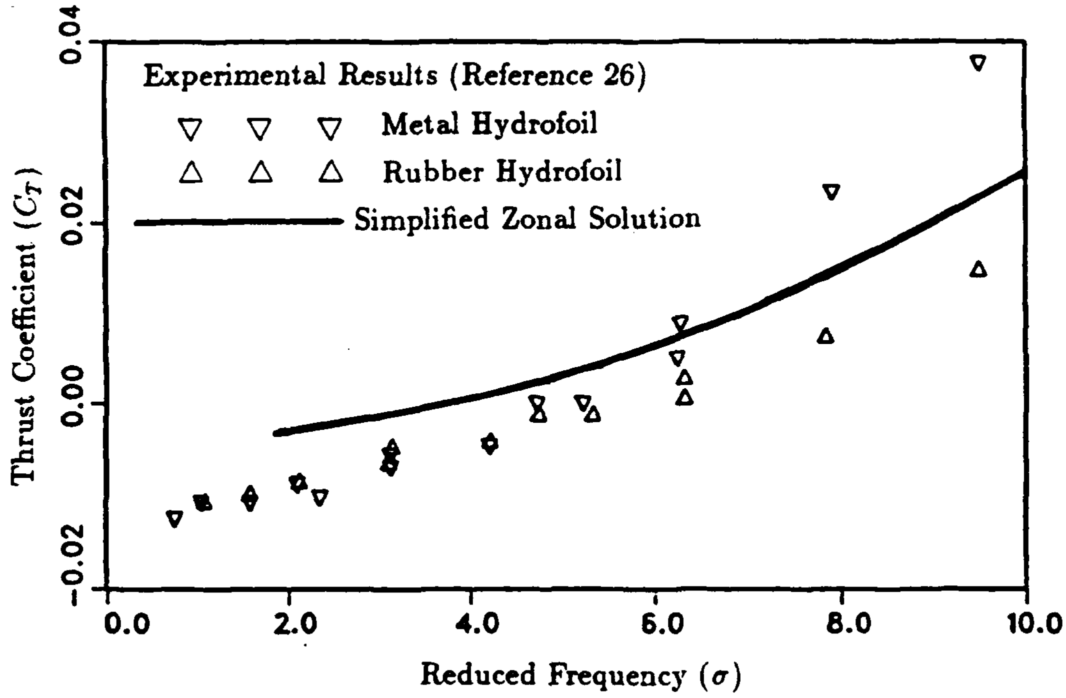
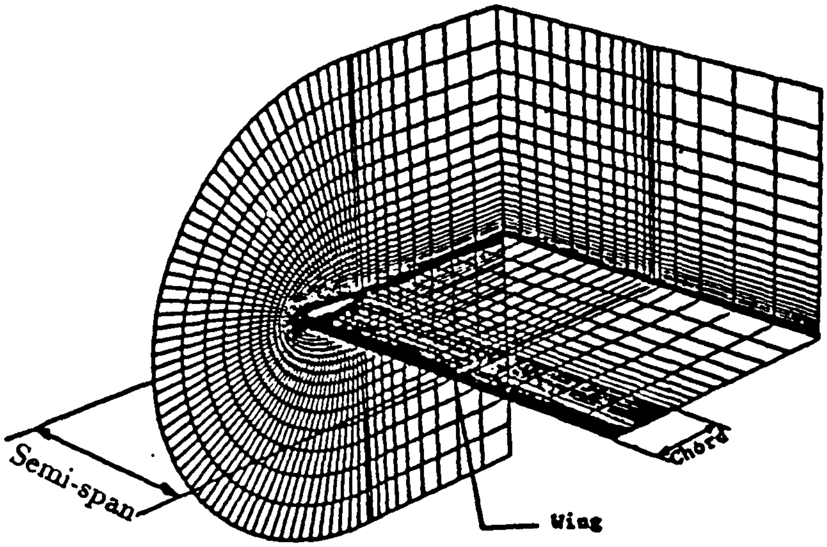
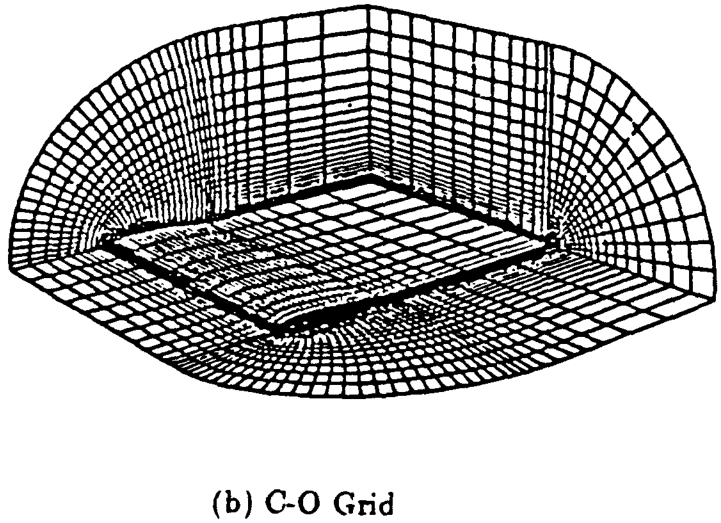


Figure 11. Thrust Coefficient for Flexible Hydrofoils

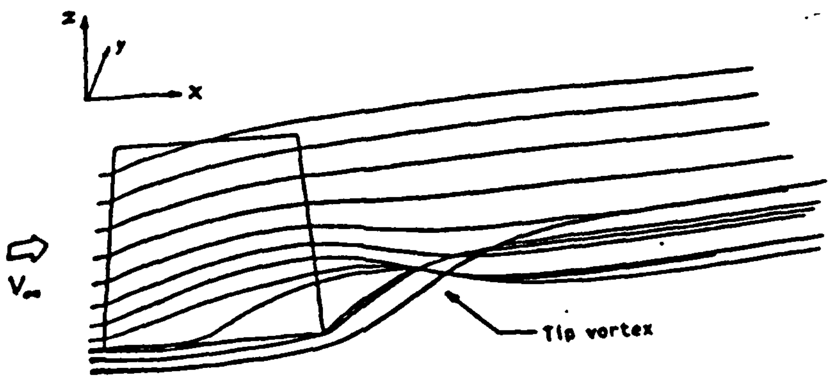


(a) C-H Grid

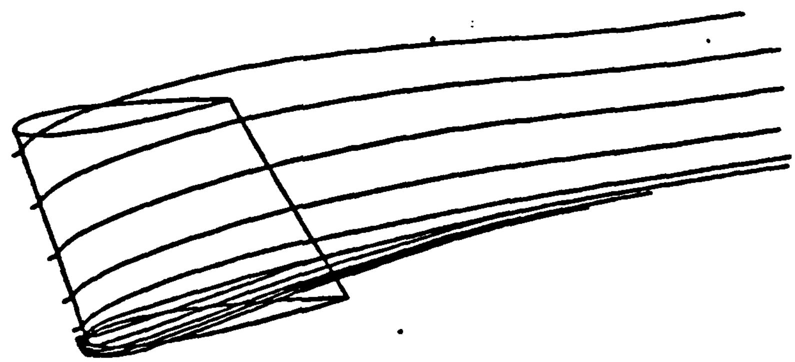


(b) C-O Grid

Figure 12. Grid Systems around an NACA0012 Wing

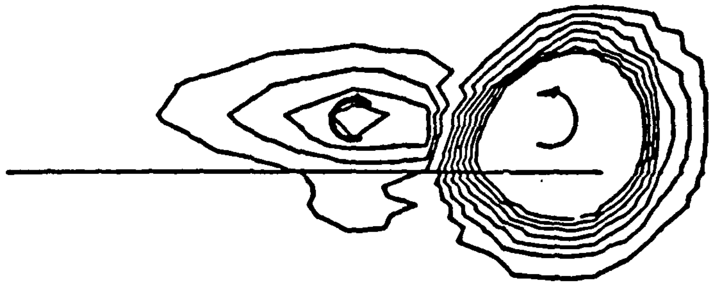


(a) Flat Plate Wing

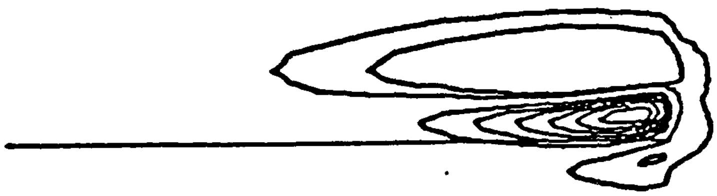


(b) NACA0012 Wing

Figure 13. Streamlines over Wings



(a) Flat Plate Wing



(b) NACA0012 Wing

Figure 14. Vorticity Contours in Wake of Wings

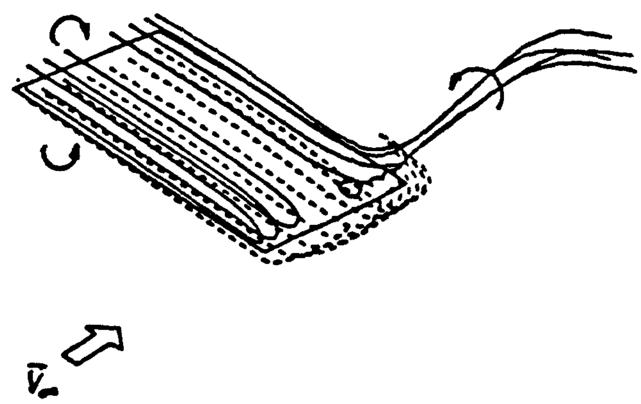


Figure 15. Vortex Lines over the Flat Plate

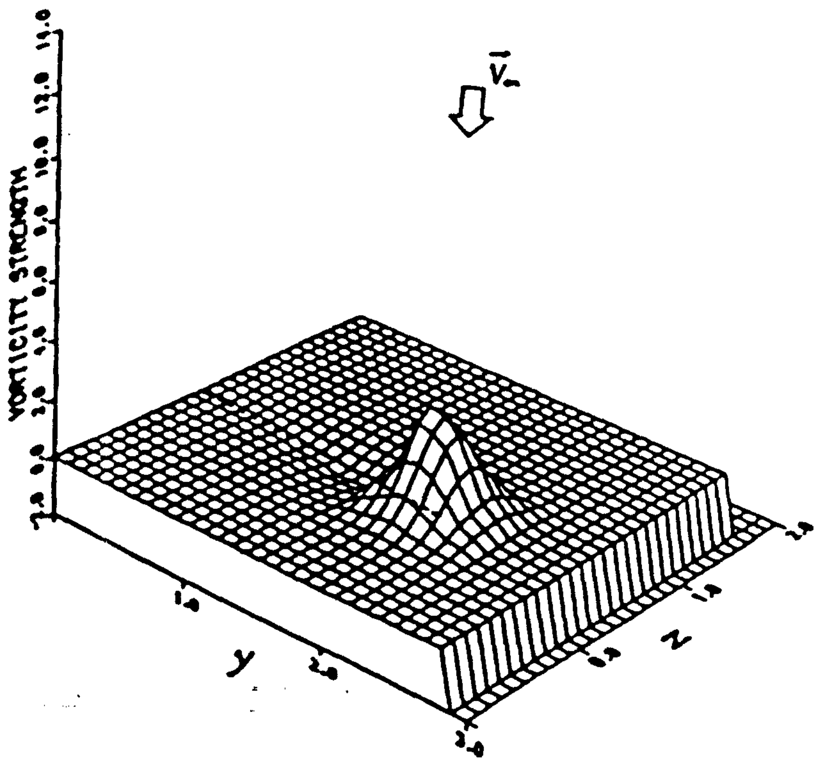


Figure 16. Vorticity Surface in Wake Plane of Flat Plate Wing